The Use of a Discrete Transistor Amplifier to Produce Active Metamaterials

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April 21, 2009

Duke University
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Senior Honors Thesis
Spring 2009

Abstract

In recent years, significant research has been done showing that artificial electromagnetic materials (metamaterials) can be built that satisfy unique characteristics in the reflection, transmission, and absorption of electromagnetic waves [1]. The commonly known metamaterials, split-ring resonators (SRRs) and electric-field-coupled resonators (ELCs), can indeed produce dynamic effects at a resonant frequency. The purpose of this research was to develop a system with a discrete transistor amplifier capable of amplifying the inductive current produced in one metamaterial and injecting this current into a second metamaterial, thereby increasing the response of the system. I have shown that through the development of impedance matching networks, it is possible to create an amplification system that will increase the magnetic response of the overall system.

I. Introduction

Recent studies have shown that artificial electromagnetic materials (metamaterials), such as split-ring resonators (SRRs) and electric-field-coupled resonators (ELCs), can be used to change electromagnetic parameters (i.e. permittivity ε and permeability μ) [1]. It is these parameters that affect the transmission of electromagnetic radiation through the material. For example, by changing the relative permeability of a material from that of free space, μ_r=1, to a bigger, positive value, one can create an opaque material, which will not allow for transmission through it [2].

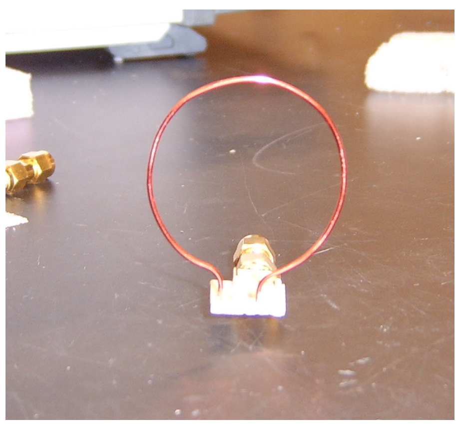

Figure 1: Picture of a Split Ring Resonator.

For this project, I worked exclusively with split-ring resonators or SRRs (see Fig. 1 above). SRRs can be modeled as the parallel RLC circuit shown below (Fig. 2). The resistance and inductance is innate to the wire itself as well as the shape of the wire, while the capacitance is created by the split or gap between the ends of the ring. Just like an RLC circuit made of lumped components, a SRR has a resonant frequency, at which the impedances of capacitor and the inductor will cancel themselves out leaving just the real component of the impedance of the wire. It is at this resonant frequency at which the SRR will operate as an opaque material, limiting transmission. Opacity occurs at the resonant frequency, because μ_r is governed by the equation below. Because the shape and size of the loop will not change nor the external magnetic field, the area of the loop (A) and the induced Emf (V_{ind}) will not change. However, the current will increase because, at resonance, the impedance will decrease to just the real, resistive component. In this way, μ_r will increase proportionally (Eqn. 1) thus changing the properties of the material.

$$\mu_r \propto \frac{I \cdot A}{V_{ind}} \tag{1}$$

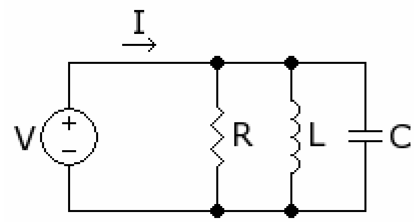

Figure 2: A SRR can be modeled as a parallel RLC circuit.

Given that μ_r increases as the current through the material increases, it becomes a motivation to increase the current through the material so as to increase this magnetic response. One way to increase this current is to make use of an active electronic component, such as an amplifier. These active components make use of an external power source to amplify the inductive currents generated by an input SRR and inject this amplified current into a second output SRR. However, active metamaterials come with many implications and therefore, I spent much of my research this semester exploring, simulating, and characterizing amplifiers as well as learning about and developing matching networks to match impedance between the SRRs and the amplifier.

II. Procedure

1. Amplifier Design

When choosing an amplifier, the goal was to utilize simple components, such as basic transistors and passive components, so that the amplifier circuit could be adapted to a small, inexpensive package. As opposed to using an operational amplifier or some other amplifier IC, it was desirable to explore a basic amplifier circuit made using only a transistor and passive components. With such a circuit, it would be easier to determine how or why the system is behaving in such a way, as all the components would be known and characterized. Given this, it became clear that an amplifier circuit made using an NPN bipolar-junction transistor (BJT) would be the best solution. Amplifiers made with BJTs are simple in principle as well as inexpensive. They also allow the amplifier to run off of one power source, which, again, is convenient for simplifying the operation of the design, making it more practical. This research was directed at microwave radiation, primarily around 1 GHz, which initially posed some concerns regarding the BJT. However, the standard silicon BJT manufactured today has a cut-off frequency of up to 10 GHz, and therefore was ample for the purposes of this research [4]. Next, it was necessary to determine what configuration to operate the BJT amplifier circuit in: common base or common emitter (Fig. 3).

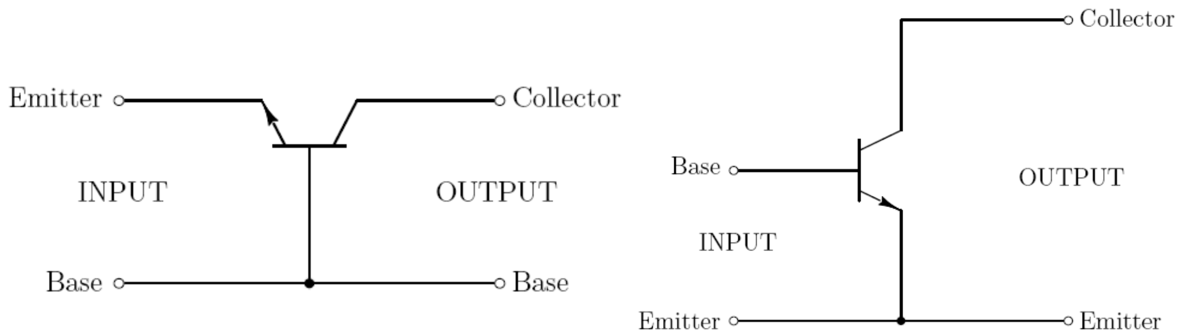

Figure 3: a) The common base configuration. b) The common emitter configuration.

It seemed interesting to consider other configurations besides the most typical used, common emitter configuration [3]. For example, the common emitter configuration offers high voltage and high current gain, while the common base configuration offers just high voltage gain. While it was interesting to consider the various types of configurations, the common emitter made the most sense since it offered both voltage and current gain, and, being the most popular circuit, had the most information readily available on how to set up the passive components. Now, that the configuration and the type of transistor was chosen, I began to simulate various amplifier circuits in the circuit simulation program, LT Spice. An example of the circuit I created and simulated can be seen below in Figure 4. In the schematic below, R5, R6, C2, C3, L4, and L5 represent the theoretical values of the resistance, capacitance, and inductance in the SRR. In order to find these values as well as the theoretical inductive EMF of the loop, I wrote a MATLAB script that calculated these values based on the size of the loop of wire, the strength of the external magnetic field, and the desired resonant frequency which the SRR will operate at (see Appendix 1 for script).

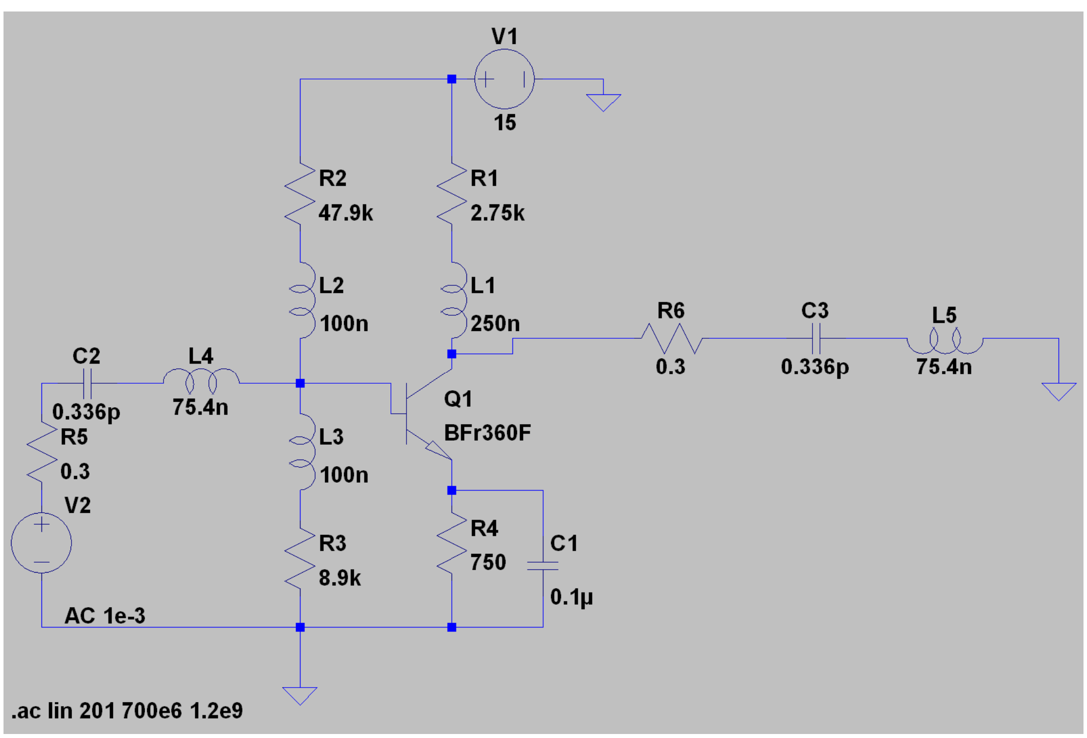

Figure 4: An example amplifier using an NPN BJT in the common emitter configuration.

However, the number of components (RF chokes, resistor values, DC blocking capacitors, etc.) necessary to design a high gain, stable amplifier at microwave frequencies, involved a great deal of optimization that was really secondary to the true purpose of the research: to incorporate an amplifier circuit into a system of SRRs to increase the system magnetic response. Therefore, it made sense to buy an evaluation amplifier board, which could be used to test different matching networks and SRRs without having to solder and desolder wires and components. The NE662M04-EVGA09 Evaluation Board was selected, which is manufactured by California Eastern Laboratories (the schematic is below in Figure 5). This particular evaluation board was a basic amplifier circuit featuring a single NPN BJT (NE662M04) and just passive components. It offered a 20 dB small signal gain (fairly high gain for a single BJT) and was optimized for 900MHz, operating with a single 3V power source [4]. This board worked extremely well for the purposes of this project.

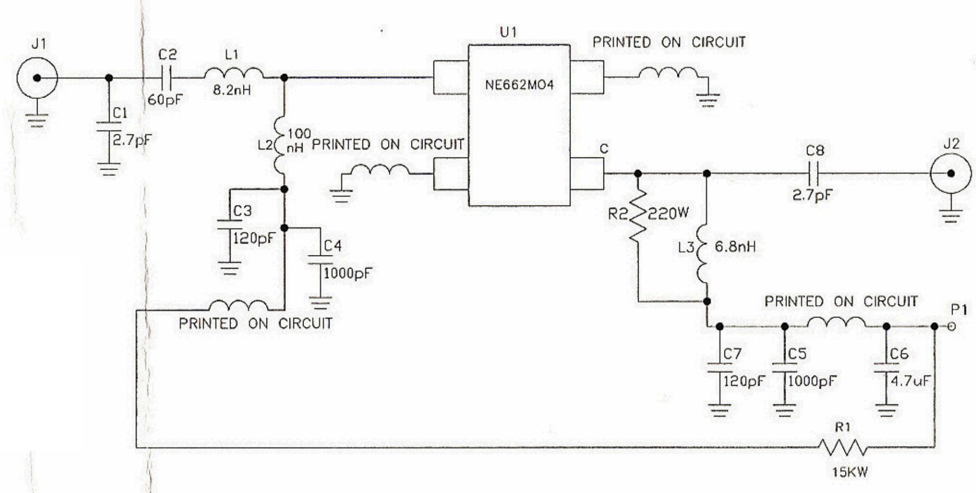

Figure 5: Circuit schematic of the NE662M04-EVGA09 Evaluation Board.

2. Amplifier and SRR Characterization

In order to determine the matching networks of the system shown below in Figure 6, I needed to determine the input and output impedances of the evaluation board amplifier as well as the impedances of the source and load loops at various frequencies.

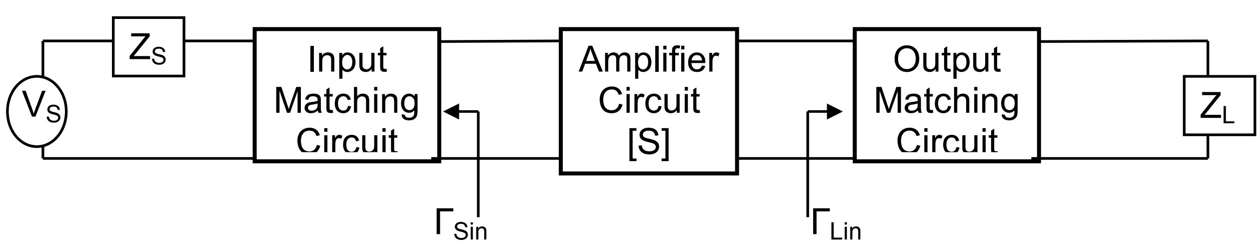

Figure 6: Diagram of system, where Z_S and Z_L represent the SRR loops.

To accomplish this I used a procedure laid out in the textbook, Microwave Engineering, by David M. Pozar [4]. First, the full S-parameters (S11, S12, S21, and S22) of the amplifier circuit are determined over the range of desired operational frequencies. One can determine and collect these S-parameters using a network analyzer (Fig. 7). It is important to fully calibrate the network analyzer for a 2 port system, by connecting each port (Port 1 and Port 2) to an open, a short, and a load termination to determine the calibration reflection coefficients (S11 and S22). Next, the calibration transmission coefficients are determined by connecting both ports directly together to form a thru (S21 and S12). With these S-parameters one can now find the reflection coefficients of the amplifier circuit, Γ_{Sin} and Γ_{Lin}, according to the equations below.

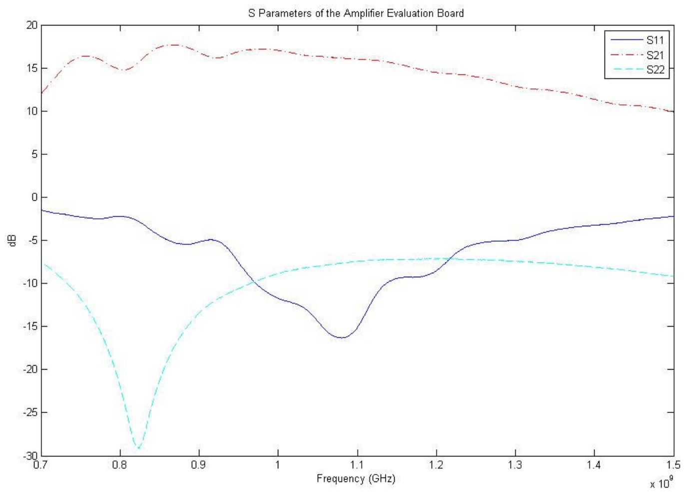

Figure 7: S-parameters of the amp, note the 15dB gain in S21 showing amp works.

$$\Gamma_{Sin} = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \tag{2}$$

$$\Gamma_{Lin} = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C2} \tag{3}$$

$$\begin{aligned} B_1 &= 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ B_2 &= 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_1 &= S_{11} - \Delta \cdot S_{22}^* \\ C_2 &= S_{22} - \Delta \cdot S_{11}^* \\ \Delta &= S_{11}S_{22} - S_{12}S_{21} \end{aligned} \tag{4}$$

Once determining these reflection coefficients, Γ_{Sin} and Γ_{Lin} can be converted the into impedances, Z_{Sin} and Z_{Lin}, using an algebraic form of the standard reflection coefficient equation (both are shown below).

$$\Gamma_{Lin} = \frac{Z_{Lin} - Z_0}{Z_{Lin} + Z_0} = \frac{\frac{Z_{Lin}}{Z_0} - 1}{\frac{Z_{Lin}}{Z_0} + 1} \tag{5}$$

$$R_{Lin} + jX_{Lin} = Z_0 \cdot \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

In the above equations, Γ_r and Γ_i are the real and imaginary components of the reflection coefficient, Γ_{Lin}, respectively. This method yields Z_{Lin}, the input impedance to the load matching network, and it is imperative for determining the values of the lumped components in the matching network. This same exact method can also be used to calculate Z_{Sin} (see the script Sparamscrip3.m in the Appendix).

Once the input impedances to the load and source matching networks have both been calculated, it is necessary to determine the real, experimental impedance of the SRR, as opposed to just a theoretical value, which I demonstrated I can calculate simply by knowing the size of the loop and the desired resonant frequency. The method for determining the experimental impedance of the SRR at various frequencies is quite similar to the one used above to calculate the input impedances produced by the amplifier. Given S11 is the reflection coefficient, Γ_S, the impedance of the loop, Z_S, can be calculated directly, exactly as done in Equation 5 above (see Fig. 8). And with the experimentally determined input impedances form the amplifier as well as those of the SRRs, the matching networks can now be determined.

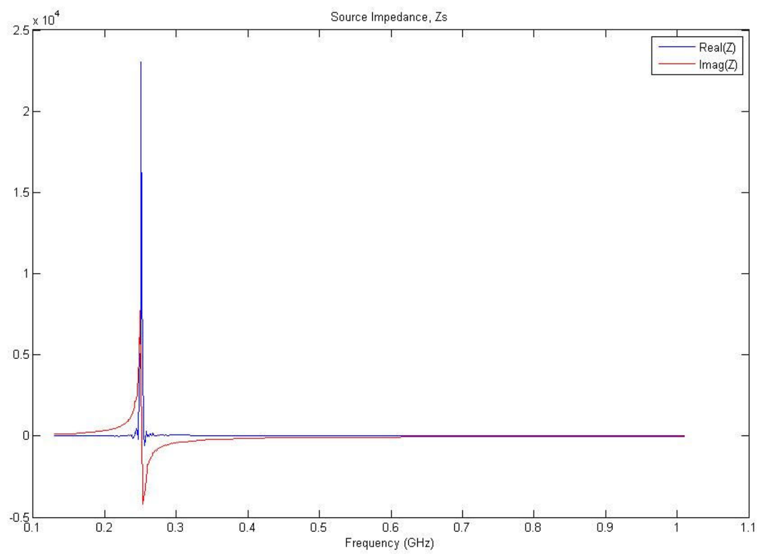

Figure 8: The source SRR impedance, note the resonance at 280 MHz.

3. Impedance Matching Networks

Microwave Engineering books spend a great deal of time discussing matching using stubs. The advantage of using stubs is that they can operate at very high frequencies, such as greater than 1 GHz, at which point the wavelength of the wave is on the order of centimeters. Generally, the length of traces on a circuit board must be less than one-tenth a wavelength or else the RF signal is susceptible to large amounts of noise. Although, it is possible, to make circuits with traces up to one-fourth a wavelength, with careful tracing techniques. The criteria on the trace size is a limitation for using lumped matching components, such as typical capacitors and inductors. However, given the small size that surface-mount capacitors and inductors can be built to today, it is possible to use lumped components in matching networks. Because, at a frequency of 1 GHz and assuming an index of refraction approximately equal to 2, the wavelength is going to be about 14cm, (λ=c/(f*n)). At this frequency, a tenth of a wavelength is over 1cm, and the matching circuits can be soldered easily with traces this small. Therefore, given the great flexibility and interchangeability of lumped components, I chose to match my SRRs using lumped component circuits.

Given the decision to use lumped components in the matching networks, there is one basic way to design the matching network, called the L Network. As can be seen below in Figure 9, the L Network consists of some reactance (X) connected in an L shape to some susceptance (B). This L network is then placed between the load and the input impedance of the amp [4].

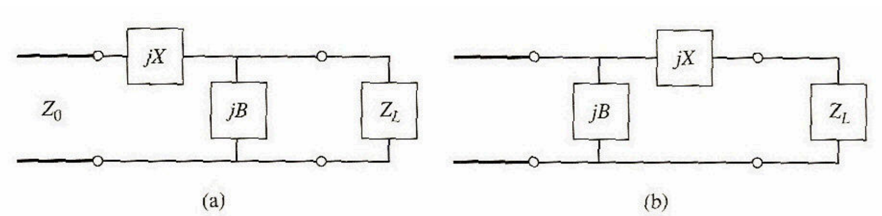

Figure 9: The L Network matching network for use with lumped elements in two configurations.

The configuration in Fig. 9a is supposed to be designed for cases where the normalized Z_L, or Z_L/Z_0, is inside the 1+jX circle on the Smith chart, implying that R_L, the real component of Z_L, is greater than Z_0. However, in this case, Z_0 is really, Z_{Lin}, or the impedance generated from the amp looking into the matching network and is complex. Correspondingly, the configuration in Fig. 7b is for a normalized Z_L outside the 1+jX circle on the Smith chart, implying that R_L is less than Z_0. From dividing Z_L/Z_{Lin}, I determined that both the source and the load have normalized impedances outside the 1+jX circle, consequently meaning the real components of their impedances were less than Z_{Sin} and Z_{Lin}, respectively. This makes sense since the amplifier evaluation board should be matched to around 50Ω, while both the SRRs had impedances of around 5-30j at 900MHz, which has a real component far less than 50Ω. Nevertheless, I solved both of these configurations and simulated the results. I will show how I solved for the matching components for the configuration in Fig. 9b, since that is the ultimate choice I made for matching network.

In order to solve to B and X in the circuit above, it is necessary to create a system of equations of at least two equations to solve for the two unknowns. This can be done by writing an equation describing the equivalency of the impedances in the circuit (Eqn. 6). Next, rearrange and separate the real and imaginary parts, in order to obtain an equation describing the equivalency of the real components and the equivalency of the imaginary components (Eqns. 7 and 8). This process yields the system of equations needed to solve for the two unknowns, B and X [4].

$$\frac{1}{Z_{in}} = jB + \frac{1}{R_L + j(X + X_L)} \tag{6}$$

$$R_L = -BX_{in}R_L - BR_{in}(X_L + X) + R_{in} \tag{7}$$

$$j(X_L + X) = jBR_{in}R_L - jBX_{in}(X_L + X) + jX_{in} \tag{8}$$

Now, using any sort of algebraic solver, such as Maple, the analytical equations describing X and B can be found. I took these rather intricate analytical equations from Maple and wrote a function in MATLAB that will generate the solutions for X and B given four input parameters: X_{in}, R_{in}, R_L, and X_L (see Appendix 2 for the function). I then pass this result into another script (Appendix 3) that calculates whether X and B should be capacitors or inductors depending on their sign and then calculates the value for C or L given the desired resonant frequency, f, and the value X or B itself. For example, if X is negative that means the component should be a capacitor equal to:

$$C = \frac{|X|}{2\pi f} \tag{9}$$

This process is the exact same for the source and the load. Despite, the two configurations which yield two different matching functions, once the values X and B are determined from the function, the method to determine C and L from X and B is exactly the same, regardless of the configuration of choice. Finally, given the nature of the system of equations, there are two solutions (one for X1 and B1 and one for X2 and B2). Given the choice between these two pairs of solutions, it is generally better to pick the pair of X and B that is smaller in magnitude. In general, the smaller in magnitude pair is the best choice, because the bandwidth of the match might be better. However, depending on the values of the lump elements available, it might be better to match to higher value if there is no other choice.

Ultimately, however, this matching network is a narrow-band match. While the result should give the maximum possible gain from the amplifier or greatest system response, it is highly dependent on frequency and this gain will fall very quickly. This approach is particularly narrow-band, because not only are the values of the lumped elements in the matching network dependent on frequency, but the impedances of the source loop, load loop, and even the input impedances from the amplifier circuit are dependent on frequency. Such a system is incredibly sensitive to frequency as well as small changes in the values of the matching components.

4. Mutual Inductance

One of my concerns before testing the system experimentally is that there would be significant coupling between the SRRs due to their mutual inductance. For this reason, I wanted to attempt to model the mutual inductance between the two loops, L_{12}, and then use ADS to simulate what effects this would have on the system. Once calculating L_{12}, I could also calculate a capacitance which could cancel out the mutual inductance and determine what effects it had on the system. Mutual inductance, L_{12}, adheres to formula [5]:

$$L_{12} = \frac{N_2 \cdot N_1 \cdot \Psi_{12}}{I_1} \tag{10}$$

In this formula, N_2 and N_1 both equal the number of turns of in the loop of wire, in this case 1 for both, while Ψ_{12} is the flux from the magnetic field of the first loop through the second loop. And I_1 of course is the current through loop 1. Given all of these parameters can be determined, a script can be produced to calculate a theoretical value for L_{12} (see mutualinductance.m in the

Appendix). I had been modeling I_1 and Ψ_{12} using sinusoids and therefore, L_{12} behaved in a sinusoidal pattern. I decided to take the average to the absolute value of some values of L_{12} just to get an approximate value, which I could use to model with in ADS. I determined the theoretical value of L_{12} to be 2.23nH. Therefore, the parallel capacitance needed to cancel out this mutual inductance is equal to:

$$C = \frac{1}{\omega_0^2 * L_{12}} \tag{11}$$

At a resonant frequency, or ω_0, of 900MHz, the value of C needed to cancel out this inductance is 14.03pF. Now, these values can be used in simulations in ADS to attempt to predict the behavior the system due to mutual inductance.

III. Results

1. Simulation Results

While I used LT Spice to test the amplifier circuits as well as to simulate the circuit on the evaluation board, I mainly used the software package called ADS to test the performance of the matching circuits I determined from the analysis above. ADS has several features which make it the ideal package to use for testing matching circuits. Most importantly, ADS allows one to see the results of the match in terms of the S-parameters. In this way, the results are reported just as the network analyzer reports them, in terms of the dBs of the S-parameters. It is easy to view S21 and S11 as well as S22 and see how effective the matching networks are at allowing for transmission while diminishing the reflections at the input and output port. Also, with ADS, the input and output can be represented by terminals, where the impedance can be changed from the typical value of 50Ω to anything that is desirable, including complex numbers. I was able to insert the exact impedances of the loops into these terminals, including their complex values.

I tested 5 circuits using ADS; four different matching network combinations as well as just the amplifier circuit connected to terminals with the loop impedances and no match networks. Table 1 below contains the summary of all the different values used in the matching simulations, as well as what matching configuration was used to calculate and test them, the configuration of 9a or the configuration of 9b. In this table, the abbreviation XS stands for the lumped element used for the reactance value, X, on the source side, while, for example, BL represents the lumped element used for the susceptance, B, on the load side, etc.

Table 1: Summary of the matching networks tested in ADS.

Match Network	XS	BS	XL	BL	Configuration
1	L=3.3408nH	L=13.354nH	0.30727nH	4.2570nH	9b for Source and Load
2	L=7.3458nH	C=23.030pF	L=10.097nH	C=5.1162pF	9b for Source and Load
3	C=3.6925pF	L=2.9854nH	L=10.097nH	C=5.1162pF	9a for Source and 9b for Load

4	C=3.6925pF	L=2.9854nH	C=18.603pF	L=4.4829nH	9a for Source and Load

Figure 10 below is the circuit schematic for the system with matching network no. 2, where both the source and the load are in the configuration of Fig. 9b. Figure 11 below is the circuit schematic for the system with matching network no. 4, where both the source and the load are in the configuration of Fig. 9a. Note that the resistors with boxes around them represent the two ports of the system, and the two passive components closest two them make up the matching network for the source and the load. All the other passive components are part of the amplifier evaluation board.

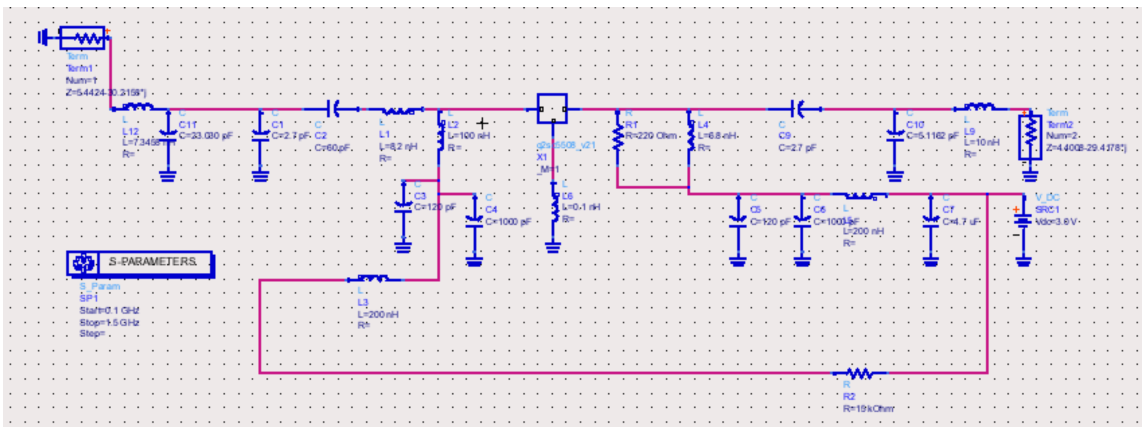

Figure 10: Circuit schematic for system with MN 2, corresponds to configuration in Fig. 7b above.

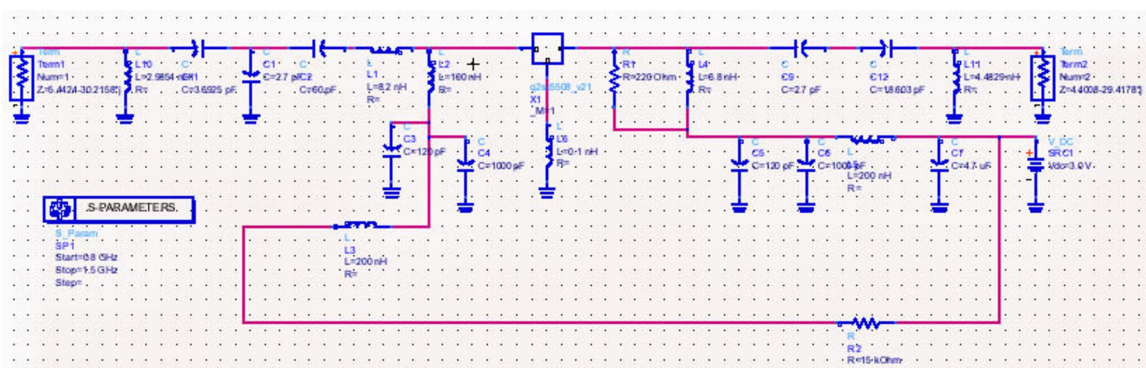

Figure 11: Circuit schematic for the system with MN 4, corresponds to configuration in Fig. 7b above..

Figure 12 below shows the simulation plots of all four matching networks, while Figure 13 shows the simulation plot for just the amplifier circuit with no matching networks between the load and source terminals and the amplifier.

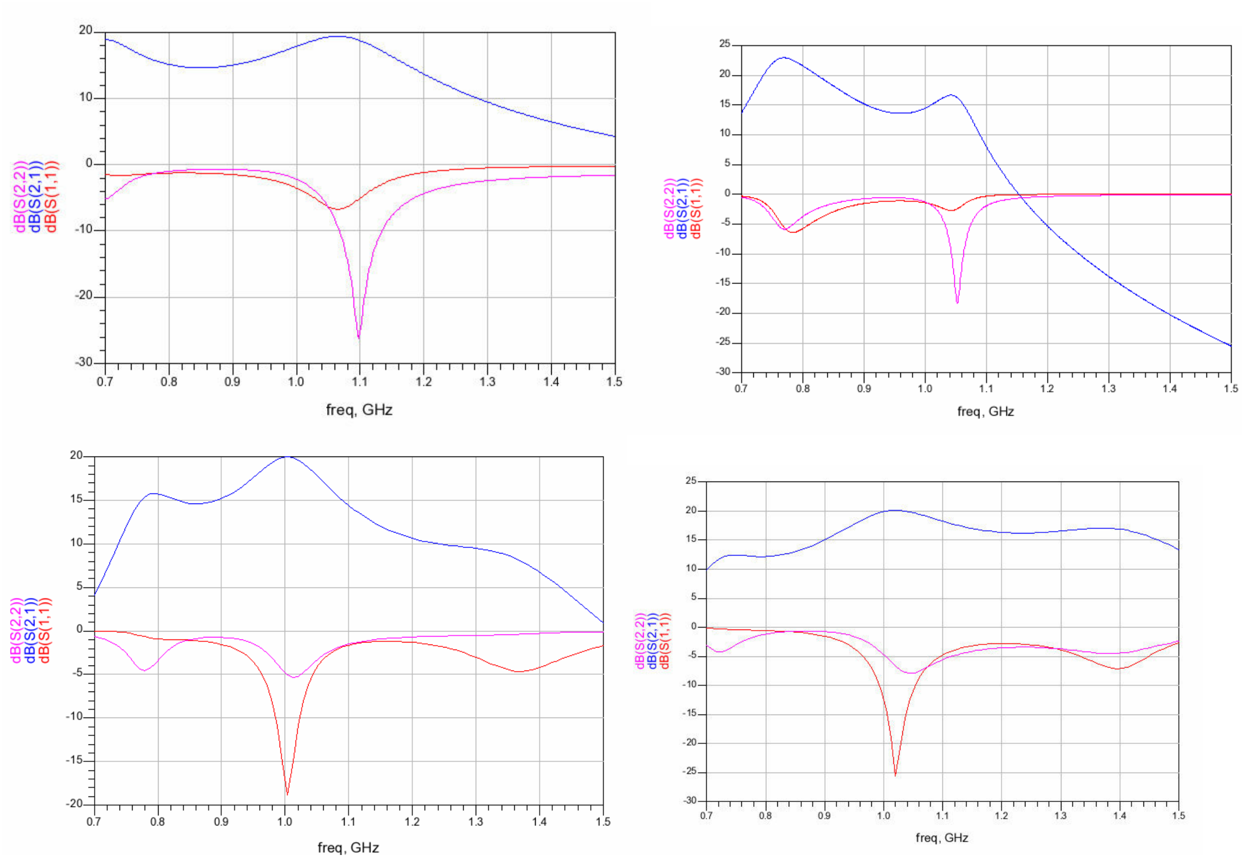

Figure 12: From top left, going clockwise, Match Network 1, Match Network 2, Match Network 3, Match Network 4.

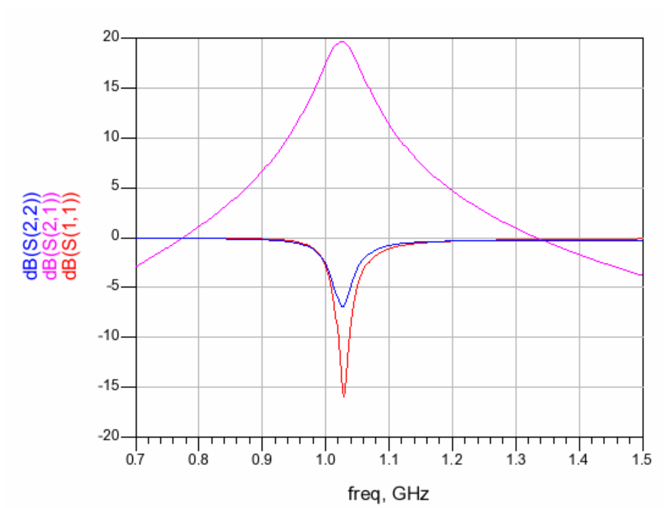

Figure 13: Simulation with no matching networks in the system.

From these simulation results, it can be seen that the matching networks, while not necessarily increasing the maximum possible gain produced by the system, does increase the bandwidth of gain. It also appears that the matching networks help especially to decrease the effects of the reflection coefficients (both S_{11} and S_{22}). However, the matching networks are supposed to be a narrow band match and they appear here as a broadband match, while the simulation with no matching network looks like a narrow band match. I am not sure why this is and because of this the simulations are not particularly helpful, so I went ahead and found experimental results.

After solving for the theoretical mutual inductance, L_{12}, as discussed above, I implemented them in my ADS simulations to see what effect this mutual inductance would have on the system. The mutual inductance was calculated to be 2.23nH, while the parallel capacitance needed to cancel the mutual inductance was found to be 14.03pF. The circuit including the inductor modeling the mutual inductance between the SRRs as well as the capacitance needed to cancel the mutual inductance is shown below in Figure 14.

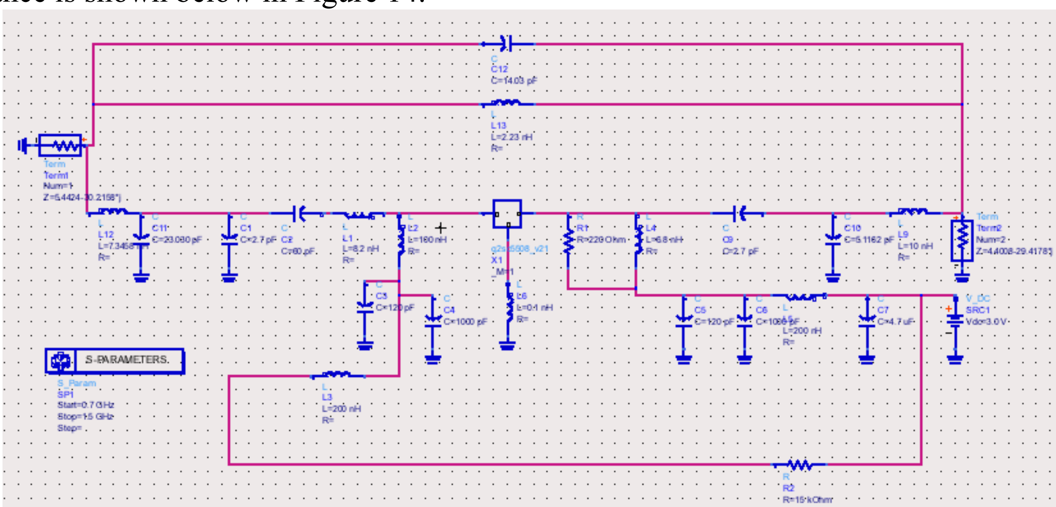

Figure 14: The system with matching network 2 as well as the mutual inductance and canceling capacitance at the top of the schematic.

Figure 15 below is a simulation of the system with matching network 2, where the plot on the left has the mutual inductance but no canceling capacitance while the plot on the right has the mutual inductance and the canceling capacitance.

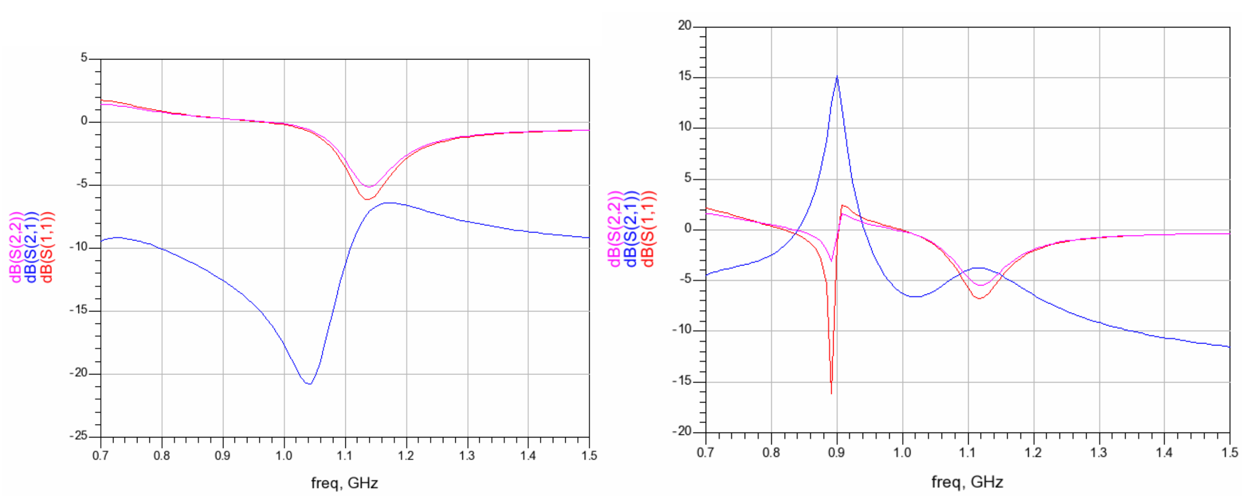

Figure 15: The system with matching network 2 left) with mutual inductance and no canceling capacitor, right) with mutual inductance and a canceling capacitor.

From the simulations above, it appears that mutual inductance can have a huge impact on the amount of signal being transmitted. Inserting a capacitor in parallel with the mutual inductance can cancel the mutual inductance and allow for transmission at the resonant frequency as seen from the plot on the right. Together, the mutual inductance and the capacitance greatly decrease the bandwidth of the gain produced from the amplifier.

Finally, given the results from the ADS simulations and taking into account mutual inductance as well as factors such as maximum transmission gain and the bandwidth of the gain, it was necessary to choose a matching network to solder and test how the system works experimentally. While there are many factors to consider, one the important factor that can be overlooked is the availability of components needed to match the circuit. This fact alone, was one of the main reasons I chose matching network 2 to solder and test. Matching network 2 performed as well as the other networks from the simulation and the lumped component values were especially convenient to values available in the lab. For these reasons, I chose to solder and test matching network two. However, as I mentioned the values for the matching component were not exact and so I had to use approximate values. The actual values I used to solder matching network no. 2 are shown below in Figure 16.

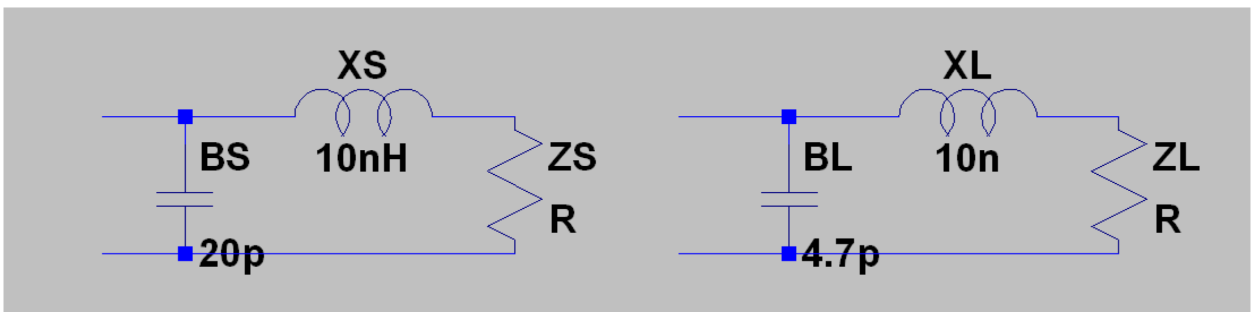

Figure 16: Actual values used to solder matching network number 2.

2. Experimental Results

After soldering the matching networks, I was ready to begin testing the system. To test the system I used a waveguide in conjunction with a network analyzer. A picture of the basic lab setup can be seen below in Figure 17.

Figure 17: Lab setup

After calibrating the network analyzer to work properly with the waveguide, I made a few basic measurements of the transmission coefficient, S_{21}, that yielded some very interesting results. First I measured S_{21} for the source SRR attached to the source matching network, to see if the MN worked properly and shifted the source up to a frequency of 900MHz. From, the plot below in Fig. 18, one can see the source MN works nearly perfectly in shifting the resonance frequency of the loop.

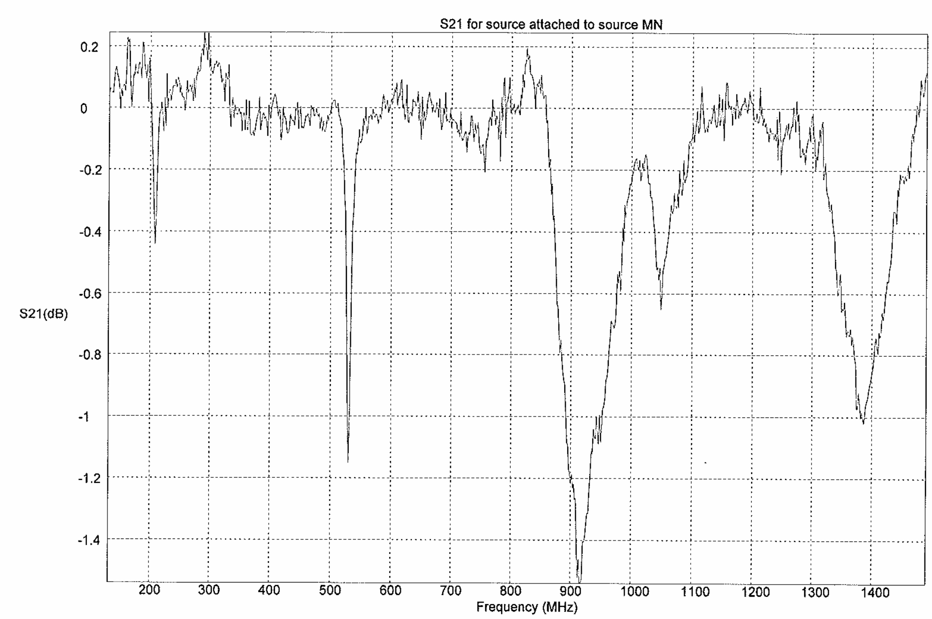

Figure 18: S21 of the source SRR connected to the source MN. The resonance is now tuned to 900MHz.

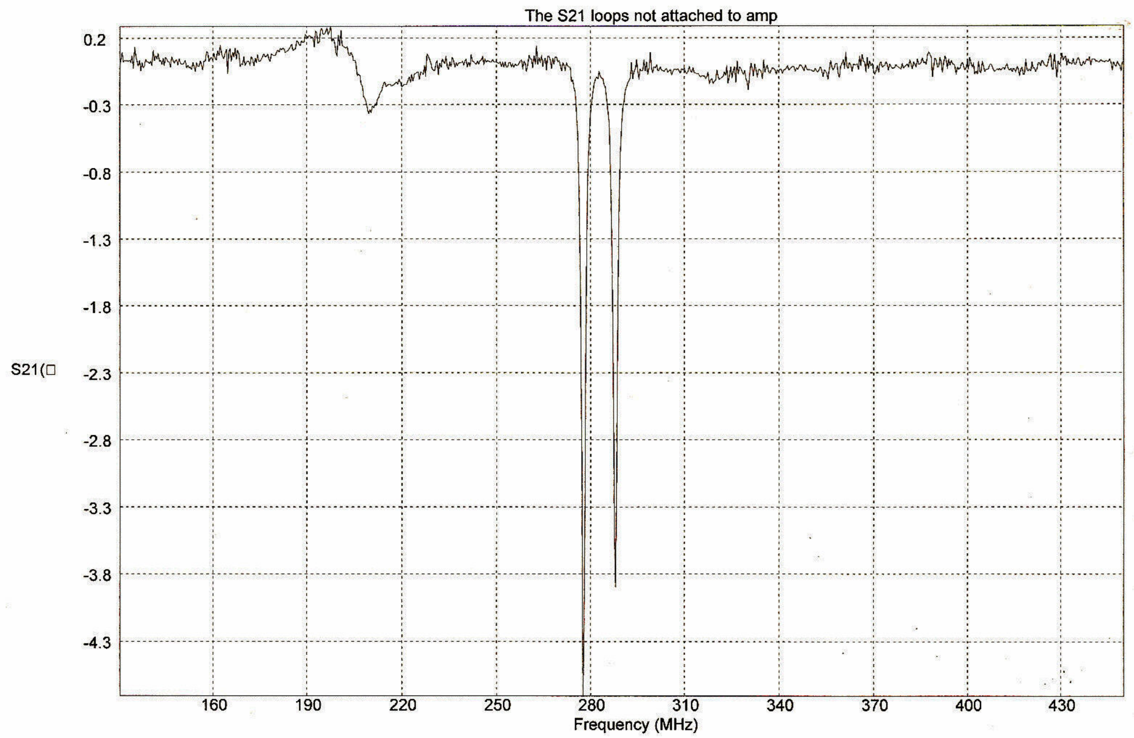

Figure 19: S21 of two SRRs in waveguide, not connected to anything else.

Contrary to what the simulation predicted regarding the effect of mutual inductance on the system, the two loops in the waveguide of Fig. 19 performed just as well as the single loop did in Fig. 18. The effects of mutual inductance did not harm the SRRs ability to reflect and block transmission at its resonance. I performed the measurement again with the two SRRs but they performed again just as well together as they did each individually. This result indicated to me that mutual inductance might not have been as big a factor as I originally thought.

Next, I hooked the two SRRs up to the amp evaluation board without the matching networks. I took measurements of S_{21} with the DC bias power to the amp both on and off (see Figs. 20 and 21), and I discovered another crucial result.

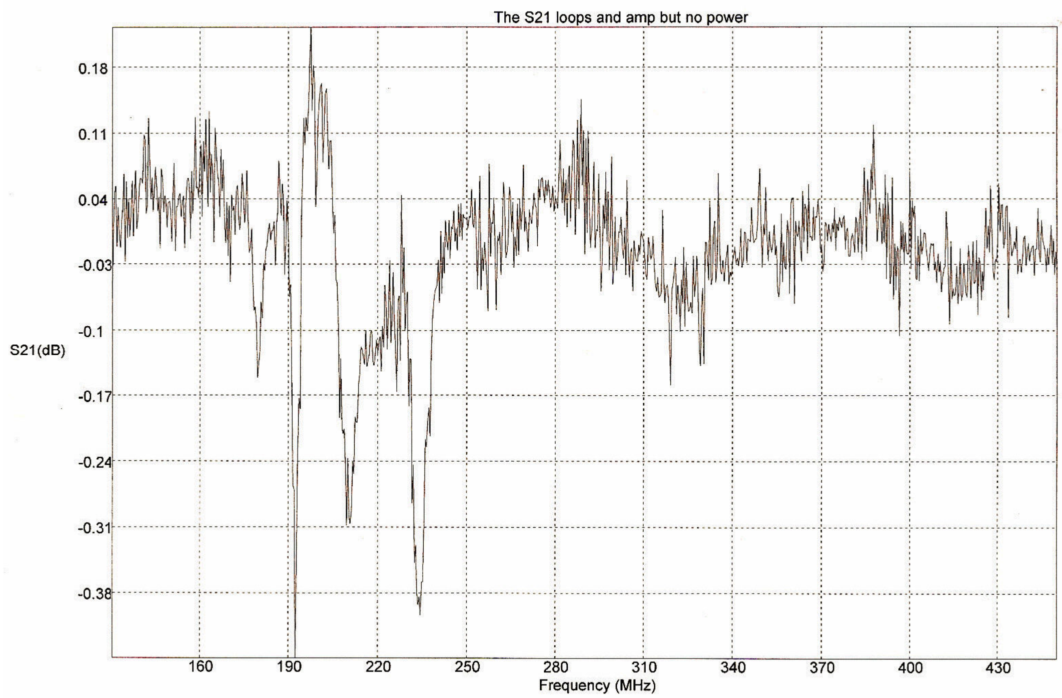

Figure 20: S21 of the two loops connected to the amp but with no power on the amp.

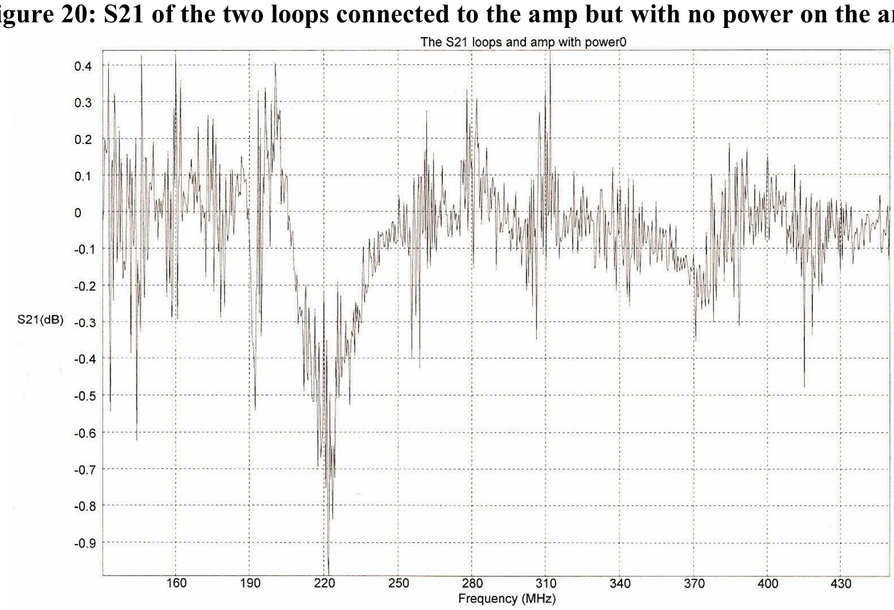

Figure 21: S21 of the loops and powered amp but no matching networks.

Unlike the simulation seemed to suggest, the impedance of the amp is too high for the SRRs to operate normally. This result shows that even with power on the amp, the SRRs can only achieve a modest, yet measurable, resonance response when attached to the amp with out matching networks. Perhaps, the amplifier has low gain at 200MHz, therefore this result stresses the necessity of using matching networks in active metamaterials to shift the frequency up to the frequency of the optimum response of the amp.

Next, I hooked up the matching networks to the loops and the amplifier, thus completing the system as it was intended. First, I measured S_{21} of the loops connected to the matching and to the amp, but with no power on the amp (see Fig. 22).

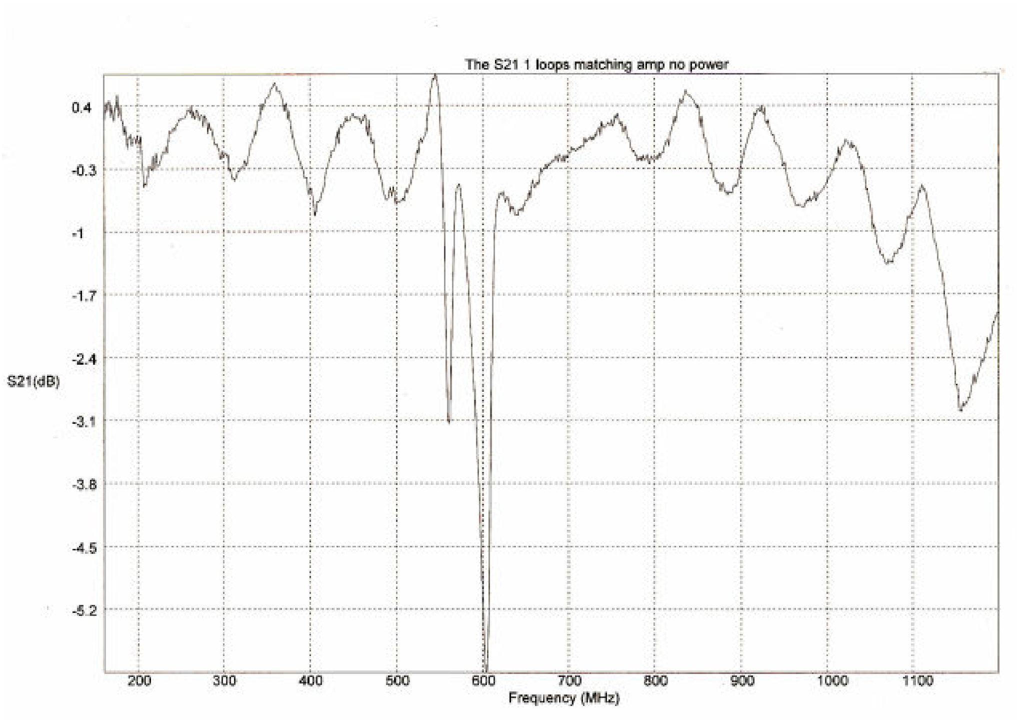

Figure 22: S21 of Loops connected to MNs, or matching networks, and amp with no power on amp.

The matching networks caused a resonant frequency shift as they were desired to do up to 600 MHz (although they were supposed to be resonant at 900 MHz). Also, the matching networks caused an increase in the response of the system, from about -4.2 dB with just the SRRs in the waveguide to a stable -5.2 dB with the SRRs attached to the matching networks and the unbiased amp. Next, I measured S_{21} with the amp power on (Fig. 23).

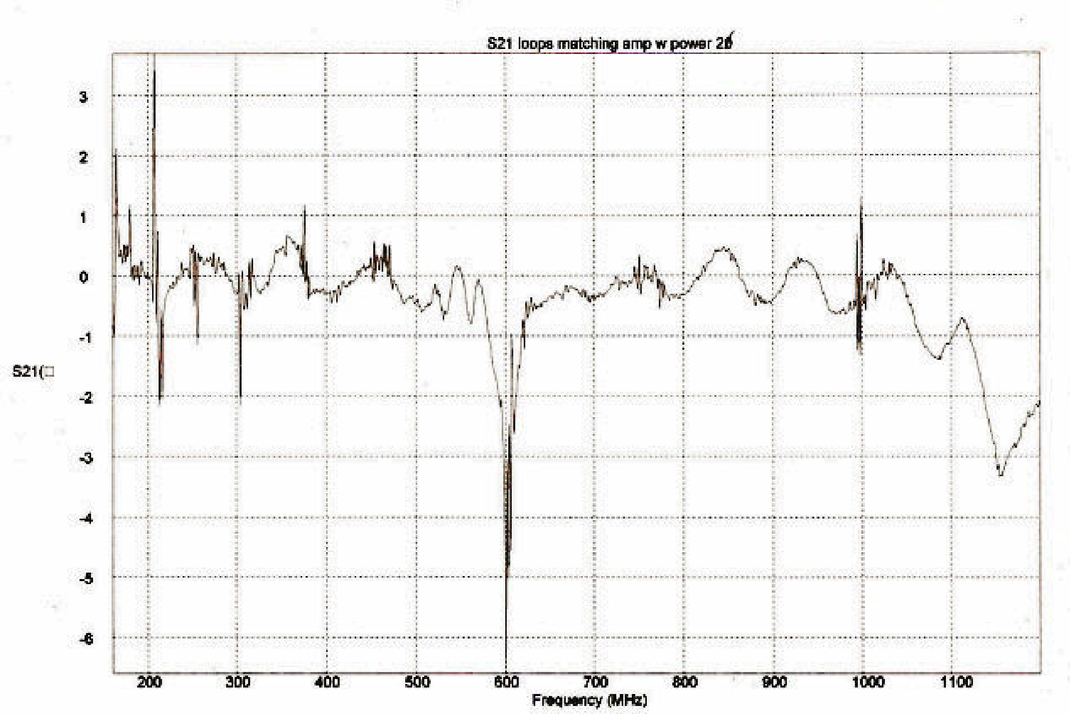

Figure 23: S21 of Loops connected to MNs and biased amp.

The biased amp created a response just as desired. It did increase the response of the system, allowing new minimum to be created for S_{21} around -7 dB. Therefore, the objective of the research was met in that the amplifier created an amplified response. However, the response was not stable with the biased amp. The S-parameter, S_{21}, would spike and drop back down, going through about a 2.5 dB with -4.5 dB as the minimum system response at 600 MHz and -7 dB as the maximum system response. Somehow, connecting the matching networks to the amplifier shifted the resonant frequency down to 600MHz, when it should be around 900MHz. The reason for this down shifting needs to be solved. Nevertheless, I believe this test proves the concept that a discrete transistor amplifier can be used to increase system response of an SRR.

Finally, I began disconnecting the system and decided to hook the SRRs directly up to the matching network itself. I connected the source SRR to the source matching network which was connected directly to the load matching network and finally the load matching network was connected to the load. I measured S_{21} and saw a very interesting result shown in Figure 24 below.

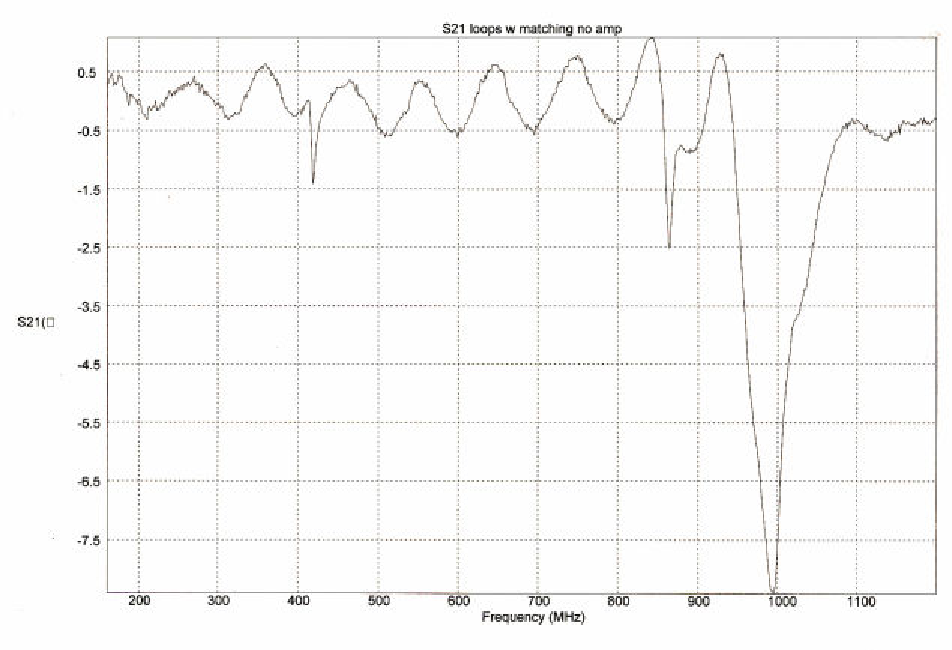

Figure 24: S21 of loops connected to MNs with no amp in between.

The matching networks connected in this configuration create a large frequency shift from the normal resonant frequency of the loops, around 280 MHz, all the way to a resonant frequency of about 1 GHz. The response is also increased as well so it is almost around -8.5 dB. It is unclear why such a drastic change in system response would occur. I removed the load matching network from the system and performed the same measurement and then replaced the load matching network and removed the source matching network and repeated the measurement. The result of these measurements for the load matching network is shown below in Fig. 25. However, these configurations of the matching networks did not create the same system response and in fact hurt the response, causing S_{21} to decrease it response from about -8.5 dB about to only about -1 dB.

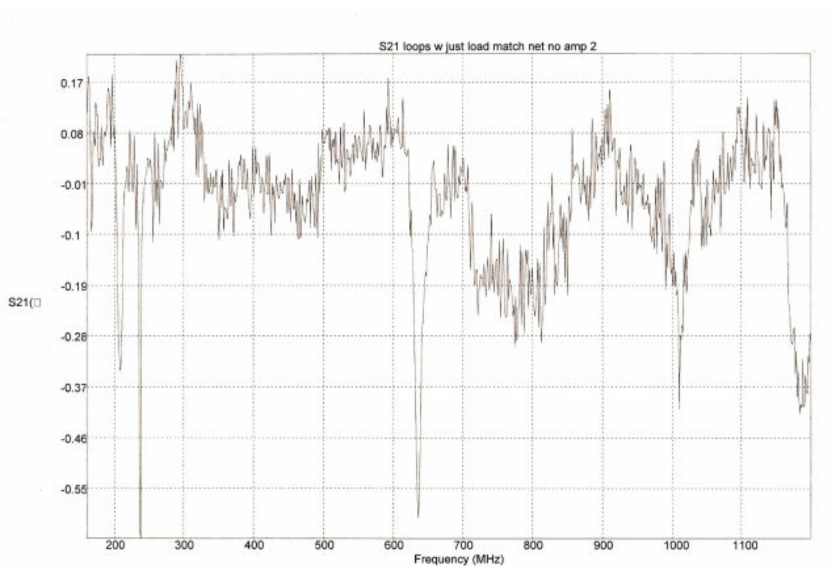

Figure 25: Loops connected to just load MN. Response not as large.

IV. Conclusions and Future Work

Ultimately, I was able to demonstrate that a discrete transistor amplifier can be used in active metamaterials to create an amplified system response. I am very happy to be able to meet the goal of that was originally laid out. Some other key results include: mutual inductance was not a large factor in disturbing the performance of the system. Also, the importance of careful matching networks was shown, as the system will not function properly with out them. I demonstrated a method for successfully creating a matching network from the measured S-parameters of amplifiers and SRRs, and using this matching network to shift the frequency of the SRR to a certain desired frequency. The ability to shift the frequency of an SRR simply using matching networks is quite a powerful result, because it would allow one to change the resonant frequency of the SRR simply by resoldering a couple of lumped components in the matching network as opposed to having to rebuild a brand new SRR each time you want to change the resonant frequency. However, from results arise more questions and areas where I would continue working in the future.

The first question I would like to explore is why the frequencies of the source and load (and their matching networks) get downshifted when connected to the amp. This is a big question and important to truly making active metamaterials practical for applications. If you want to run an application at a certain frequency, it is important to make sure the metamaterial will operate at that frequency. I would have to run more tests to solve this problem, however, I have some ideas as to how to determine what is happening in the system that causes this frequency downshift. The second question, I would like to address is why did the response of the system oscillate when the amplifier was biased but was completely flat and stable when it was unbiased. I might try sticking a capacitance across the two SRRs first, to see if this cycling is due to some sort of inductive coupling between the two SRRs, which is increased by the amplifier. Finally, I would work on developing the active metamaterial for more application based studies. For example, I would attempt to make the amplifier more compact. So I would opt to create a compact amplifier on a PCB, which would be small enough that perhaps it could be embedded in a wall or some surface. For more rigorous testing, I would also construct the SRR and the matching network out of copper traces as opposed to cutting and over-soldering small PCBs to get the surface mount components to rest properly. This would also make it easier to swap lumped components out on the matching network so the frequency of the SRR could be changed quickly. There is great potential for active metamaterials in a number fields and applications.

Acknowledgements

I would like to thank my advisor, Dr. Steven Cummer, first and foremost for taking me on into his research lab and challenging me. I would like to thank all those in Dr. Cummer's lab, especially Bogdan Popa, Thomas Hand, Mark Gu, and Lucian for always helping me when I needed it. Finally, I would also like to thank Dr. Massoud, for helping me sort out my initial questions regarding discrete transistor amplifiers.

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Appendix

Appendix 1 – inductancesim2.m

```
%Doug Bycoff
%Simulation Script 1
%1/29/09 modified March 31, 2009

%d is length of side of loop in meters
d=0.02;
mu0=4.*pi.*10^-7;
L=3.*mu0.*d; %inductance in loop

R=0.3; %Resistance of wire assumed to be about 1 ohm

f=900e6; %operating frequency is 900MHz on account of the board
w0=2.*pi.*f;

C=1./(L.*w0.^2); %capacitance of gap to cancel inductance

T=1/f; %period of wave
t=linspace(0, 10.*T);
P=0.1e-3; %power 1mW at 0dB
c=3e8;
B0=P/c; %assume magnitude to be 1mT
B=B0.*cos(w0.*t);

area=d.^2;
flux=area.*B;

%emf is negative derivative of flux with respect to time (volts)
Emf=area.*B0.*w0.*sin(w0.*t); %in volts this in small signal AC voltage
maxEmf=max(Emf)
%define impedance Z, though technically capacitor impedance should cancel
%inductance impedance


Z=R + i.*w0*L + 1./(i.*w0.*C); %impedance Z
%Zcancel=R;
Iloop=Emf./Z;
%Iloopcancel=Emf./Zcancel;

MaxI=max(abs(Iloop))
MinI=min(abs(Iloop));
AvgI=mean(abs(Iloop));

%MaxIcan=max(Iloopcancel);
%MinIcan=min(Iloopcancel);
%AvgIcan=mean(Iloopcancel);
```

Appendix 2 – Matching Functions

This function solves X and B from configuration in Fig. 9b.

```
%Matching function
%April 12, 2009
%Doug Bycoff
%equations found from solving system of equations in Maple
function [X1, B1, X2, B2]=matching(RL, XL, Rin, Xin)
B1=-(Xin*RL-sqrt(-Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))/(RL*(Rin^2+Xin^2));

X1=((RL-((Xin*RL-sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Xin)/(Rin^2+Xin^2)...
    -((Xin*RL-sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Rin*XL)/(RL*(Rin^2+Xin^2))-Rin)...
    *RL*(Rin^2+Xin^2))/((Xin*RL-sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Rin);

B2=-(Xin*RL+sqrt(-Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))/(RL*(Rin^2+Xin^2));

X2=((RL-((Xin*RL+sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Xin)/(Rin^2+Xin^2)...
    -((Xin*RL+sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Rin*XL)/(RL*(Rin^2+Xin^2))-Rin)...
    *RL*(Rin^2+Xin^2))/((Xin*RL+sqrt(-
Rin^2*RL^2+Rin^3*RL+Xin^2*RL*Rin))*Rin);
end
```

This function solves X and B from configuration in Fig. 9a.

```
%Matching function
%April 12, 2009
%Doug Bycoff
%equations found from solving system of equations in Maple
function [X1, B1, X2, B2]=matchingin(RL, XL, Rin, Xin)
B1=(XL*Rin+sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*RL*Rin))/(Rin*(RL^2+XL^2));

X1=(((Xin*(XL*Rin+sqrt(-
Rin^2*RL^2+Rin*RL^3+XL^2*RL*Rin))*RL)/(Rin*(RL^2+XL^2))...
    +(XL*(XL*Rin+sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*Rin*RL)))/(RL^2+XL^2)-
Rin+RL)*Rin*(RL^2+XL^2))...
    /((XL*Rin+sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*Rin*RL))*RL);

B2=(XL*Rin-sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*RL*Rin))/(Rin*(RL^2+XL^2));

X2=(((Xin*(XL*Rin-sqrt(-
Rin^2*RL^2+Rin*RL^3+XL^2*RL*Rin))*RL)/(Rin*(RL^2+XL^2))...
    +(XL*(XL*Rin-sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*Rin*RL)))/(RL^2+XL^2)-
Rin+RL)*Rin*(RL^2+XL^2))...
    /((XL*Rin-sqrt(-Rin^2*RL^2+Rin*RL^3+XL^2*Rin*RL))*RL);

end
```

Appendix 3 – sparamscript3.m

```
%Doug Bycoff
%April 12, 2009
%S param script for evaluating amplifier and matching

clear;
%load data
load f.mat;
load S11R.mat;
load S11I.mat;
load S12R.mat;
load S12I.mat;
load S21R.mat;
load S21I.mat;
load S22R.mat;
load S22I.mat;

%form complex S-param variables which is given in real/imaginary way
S11=S11R+j.*S11I;
S12=S12R+j.*S12I;
S21=S21R+j.*S21I;
S22=S22R+j.*S22I;

%check if amplifier is unconditionally stable
%Unconditionally stable if abs(delta)<1 and K>1 or mu>1 from Pozar, Microwave
%Engineering, 3rd ed. p. 551 and 546
delta=S11.*S22-S12.*S21;
K=(1-abs(S11).^2-abs(S22).^2+abs(delta).^2)./(2.*abs(S12.*S21));
mu=(1-abs(S11).^2)./(abs(S22-delta.*conj(S11))+abs(S12.*S21));

%check stability at frequency of choice, fchoice
fchoice=900e6;
for k=1:length(f);
    if (f(k)<=fchoice && f(k+1)>fchoice)
        x=k;
    end
end
if abs(delta(x))<1 && K(x)>1
    fprintf('Unconditionally Stable \n')
else
    fprintf('Not Unconditionally Stable \n')
end

%check if amplifier can be considered unilateral if abs(S12) is close
%enough to 0 if 1/(1+U)^2<GT/GTU<1/(1-U)^2 is a few tenths of a db then
%unilateral assumption is justified
U=(abs(S12).*abs(S21).*abs(S11).*abs(S22))./((1-abs(S11).^2).*(1-
abs(S22).^2));
GTUmin=1./(1+U).^2;
GTUmax=1./(1-U).^2;

%matching at fchoice, fchoice at x
B1=1+abs(S11(x)).^2-abs(S22(x)).^2-abs(delta(x)).^2;
B2=1+abs(S22(x)).^2-abs(S11(x)).^2-abs(delta(x)).^2;
C1=S11(x)-delta(x).*conj(S22(x));
C2=S22(x)-delta(x).*conj(S11(x));
```

```
%calculate reflection coefficients, Pozar p. 550, quadratic equation so B1/2-
sqrt can
%be plus or minus, want mag of Gamma to be less than 1
GammaS=(B1-sqrt(B1.^2-4.*abs(C1).^2))./(2.*C1);
GammaL=(B2-sqrt(B2.^2-4.*abs(C2).^2))./(2.*C2);
if abs(GammaS)>1 || abs(GammaL)>1
    fprintf('Switch plus/minus sign of GammaS or GammaL equations \n')
end

%Convert to ZS and ZL, Pozar p. 66
zsin=((1+real(GammaS))+j.*imag(GammaS))./((1-real(GammaS))-j.*imag(GammaS));
zlin=((1+real(GammaL))+j.*imag(GammaL))./((1-real(GammaL))-j.*imag(GammaL));
%define Z0, which is really the loop impedance itself
Z0=50; %5 ohms because the imaginary and real components will cancel out
ZSin=zsin.*Z0;
ZLin=zlin.*Z0;
%first match load
%Define load components
RLin=real(ZLin);
XLin=imag(ZLin);
% %paramters of loop, theoretical
% dloop=0.02; %cm
% mu0=4.*pi.*10^-7;
% Lloop=3.*mu0.*dloop; %inductance in loop
% w0=2*pi*fchoice;
% Cloop=1./(Lloop.*w0.^2); %capacitance of gap to cancel inductance
% %loop impedance deduced from loop parameters above

%Load amd impedance values from running Bogdan's S11toimpedance.m in
%conjunction with the network analyzer
%Load is loop 2 at 900MHz
RL=4.4008;
%XL=-29.4178;
XL=0;
%call matching function, because load is outside 1+jX circle on smith chart
[X1L, B1L, X2L, B2L]=matching(RL, XL, RLin, XLin)
if X2L<0
    CXL=1./(2.*pi.*fchoice.*abs(X2L))
else
    LXL=abs(X2L)./(2.*pi.*fchoice)
end
if B2L>0
    CBL=abs(B2L)./(2.*pi.*fchoice)
else
    LBL=1./(2.*pi.*fchoice.*abs(B2L))
end
%match source, loop 1
RSin=real(ZSin);
XSin=imag(ZSin);
RS=5.4424;
XS=0;
%XS=-30.2158;
%call matchingin function, cause source is inside 1+jX circle on smith chart
[X1S B1S, X2S, B2S]=matching(RS, XS, RSin, XSin)
if X2S<0
    CXS=1./(2.*pi.*fchoice.*abs(X2S))
```

```
else
    LXS=abs(X2S)./(2.*pi.*fchoice)
end
if B2S>0
    CBS=abs(B2S)./(2.*pi.*fchoice)
else
    LBS=1./(2.*pi.*fchoice.*abs(B2S))
end

%check which matching network
zlcircle=(RL+j*XL)/ZLin
zscircle=(RS+j*XS)/ZSin
```

Appendix 4 – mutualinductance.m

```
%Doug Bycoff
%April 17, 2009
%Mutual Inductance between the two wires
%Mutual Inductance, L12=N2*N1*flux12/I1

%Define parameters
N1=1; %number of turns of wire for loop 1
N2=1; %number of turns of wire for loop 2
f=900e6; %operating frequency is 900MHz on account of the board
w0=2.*pi.*f;
P=0.1e-3; %power 1mW at 0dB
c=3e8;
B0=P/c; %assume magnitude to be 1mT
d=0.035; %diameter of both loops is 3.5cm
r=d/2; %radius of loops
Z1=5.4424-30.2158*j;

%Calculate flux12 and current through loop 1
T=1/f; %period of wave
t=linspace(0, 2.*T);
B1=B0.*cos(w0.*t); %magnetic field through loop 1, it will be the same for
both loops
area1=pi*r.^2;
area2=area1;
flux1=area1.*B1;
Emf1=area1.*B0.*w0.*sin(w0.*t); %in volts this in small signal AC voltage
I1=Emf1./real(Z1);  %at resonance so ignore imaginary component of loop
flux12=area2.*B1;

%calculate L12
L12=(N2.*N1.*flux12)./I1;
absL12=abs(L12);
meanL12=mean(absL12(2:49))
Ccancel=1./(meanL12.*w0.^2)
```