Radar Signal Processing for Stand-off life sign Monitoring

by Ash-girl Chapfuwa

Advisor: Dr Jeffrey L. Krolik, Professor of Electrical and Computer Engineering

Duke University

Department of Electrical and Computer Engineering

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CONTENTS

1 Abstract

The ability to use a Doppler radar to enable remote (wireless) detection of both heart and respiration rates from chest movement could be a valuable tool in home health care and human detection. Important applications of this technology are in search and rescue (e.g. locating earthquake victims inside a building), monitoring infants wirelessly (provide non-contact alerts for sudden infant death syndrome) and elderly care. While, current technology can remotely detect respiration rate, detecting and extracting the weak heartbeat signal from the radar output (chest movement) signal remains a key technical challenge. Therefore, the purpose of this research is to develop signal processing algorithms and MATLAB displays for a software testbed radar (STRADAR), which will enable the detection and extraction of the heartbeat signal from the radar output signal. To recover the weak heartbeat signal, an approximation of the respiration signal has been developed. Five signal processing techniques that have been explored and implemented in MATLAB: blind source separation, linear prediction, time frequency analysis, frequency filtering and non-linear prediction. Non-linear prediction and blind source separation techniques have yielded the best results; heartbeat and respiration signals have been successfully separated from a simulated radar output signal. Further work is needed to recover the heartbeat signal from the experimental chest movement signal. The algorithm developed for non-linear prediction in MATLAB can be improved to achieve accurate initial bounds used in the estimation of the unknown parameters from the respiration signal model.

2 List of Abbreviations and Symbols

Table 1: Symbols used for the Radar System

A	amplitude of transmitted signal
A_e	echo amplitude
$\tau = \frac{2r}{c}$	time delay, r is the distance between target and receiver (range)
β	signal bandwidth
f_o	radar center frequency
λ_o	radar operating wavelength
$\mu = \frac{2v_\tau}{\lambda_o}$	doppler frequency , v_τ is the chest movement maximum velocity
f_b	respiration frequency (0.1- 0.8 Hz)
f_h	heartbeat frequency (0.8 - 2 Hz)
ϕ	respiration random phase
θ	transmitted signal random phase
f_s	fast time sampling frequency
t_{cpi}	slow time of a coherent processing interval

3 Introduction

3.1 Problem

Doppler radar can provide non-invasive monitoring of cardio-pulmonary activity of human subjects. Microwave Doppler radar has been used for wireless sensing of respiration rate since the 1970s [1]. Several applications for this technology are through the wall detection in emergency search and rescue, battlefield triage, remote monitoring (e.g., infants at risk of sudden infant death syndrome, burn victims, 24-hour elderly care patients in post-op, people with sleep disorders and chronically ill patients). Consequently, Doppler radar has the potential to make a significant impact in healthcare and national security.

A person sitting still has chest movements due to respiration and heartbeat. Chest movements induce a Doppler shift to the radar transmitted signal and this translates to a phase shift in the received signal. Recent advances in radar technologies have made it possible to wirelessly detect respiration rate from the frequency spectrum of the radar output signal. However, detecting the weak heartbeat signal from the radar output signal demands the use of advanced signal processing separating techniques because the respiration signal masks the heartbeat signal. Therefore, wirelessly obtaining the weak heartbeat signal using a Doppler radar can be done if the human subject holds his/her breath for at least 20 seconds. Major advances in the use of radar non-contact methods to detect the heartbeat signal from the radar output signal have been successful through the use of an additional invasive sensor [1-4].

This research project focuses on the separation of the heartbeat and the respiration signals from the radar output (chest movement) signal with the objective of obtaining both the respiration and heart rates. This approach differs from the various radar non-contact methods that have been used so far by not using any invasive sensors and applying signal processing separating algorithms in MATLAB to obtain the heartbeat signal. The ability to wirelessly detect both heart and respiration rates can extend the applications of radar technology to home use and simultaneously solve the problem of battery life from the use of invasive sensors.

The signal processing techniques investigated in this research project are: frequency filtering, blind source separation (FastICA), time frequency analysis (STFT), linear prediction, and non-linear prediction. These separating techniques were applied on the simulated data (Appendix B) and experimental data.

3.2 Background

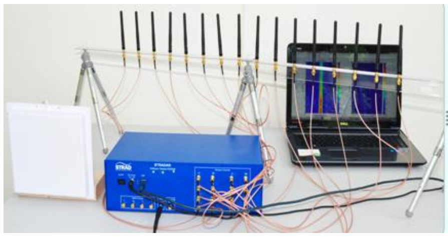


Figure 1: STRADAR system

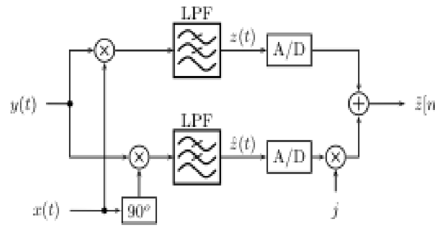


Figure 2: STRADAR signal processing

Radar System Description:

The research project is based on a functioning multiple input multiple output (MIMO) radar system with 16 element receive array and 4 transmit channels (STRADAR, Figure 1). To enable remote detection of respiration and heartbeat signals the radar device transmits *M* pulses of chirp signals every pulse repetition interval (PRI) for a duration of *M* x PRI to make a coherent processing interval (CPI). The transmitted chirp signal *x(t)* has a carrier frequency of 2.4GHz and a bandwidth of 600MHz. The target echo signal *y(t)* is received by each of the *N* elements receivers. The received signal is a phase shifted, time delayed version of the transmitted signal. Both the transmitted and received signals are processed according to Figure 2. The output *z(n)* from the radar device is sent to a digital processor (MATLAB). In MATLAB a data cube is formed from *L* samples of each PRI (fast time, *fs* = 31200Hz), *N* receiver elements and *M* pulses (slow time) and this gives an *L* x *M* x *N* matrix. Further processing of the data cube yields a Doppler spectrum that is used to determine the range and the breathing rate of the human subject (Appendix A). Determining the heart rate of the human subject requires advanced signal processing techniques that will be addressed in this research project.

Respiration Signal Model:

The waveform of the radar transmitted chirp signal,

$$x(t) = \mathrm{Re}\left(Ae^{j\left(2\pi f_o t + \frac{\pi\beta t^2}{Tr} + \theta\right)} \right) \tag{1}$$

is backscattered by the target and an echo is received by each of the *N* elements, the echo is a time delayed and phase modulated version of the transmitted signal, resulting in,

$$y(t) = \text{Re}\left(A_e e^{j\left(2\pi f_o(t-\tau)+\frac{\pi\beta(t-\tau)^2}{Tr}+\theta\right)} e^{j\mu\sin\left(2\pi f_b t_{cpi}+\phi\right)} \right) \qquad (2)$$

The radar low pass filters (LPF) the product of *y(t)* and *x(t)* before passing the result through an analog to digital converter, the product is

$$y(t)x(t) = \text{Re}\left(Ae^{j\left(2\pi f_o t+\frac{\pi\beta t^2}{Tr}\right)} A_e e^{j\left(2\pi f_o(t-\tau)+\frac{\pi\beta(t-\tau)^2}{Tr}\right)} e^{j(\mu\sin\left(2\pi f_b t_{cpi}+\phi\right)+\theta)} \right) \qquad (3)$$

Using the trigonometric identity

$$\cos(A)\cos(B) = \frac{\cos(A+B)+\cos(A-B)}{2} \qquad (4)$$

The LPF block filters out the $\frac{\cos(A+B)}{2}$ term from the product, yielding an output

$$z(t) = \text{Re}\left(0.5AA_e e^{j\left(\frac{2\pi\beta\tau t}{Tr}+2\pi f_o\tau-\frac{\pi\beta\tau^2}{Tr}\right)} e^{j(\mu\sin\left(2\pi f_b t_{cpi}+\phi\right)+\theta)} \right)$$

$$= \text{Re}\left(0.5AA_e \sum_{n=-\infty}^{\infty} J_n(\mu) e^{j\left(\frac{2\pi\beta\tau t}{Tr}+2\pi f_o\tau-\frac{\pi\beta\tau^2}{Tr}+\theta\right)} e^{j(2\pi n f_b t_{cpi}+\phi)} \right) \qquad (5)$$

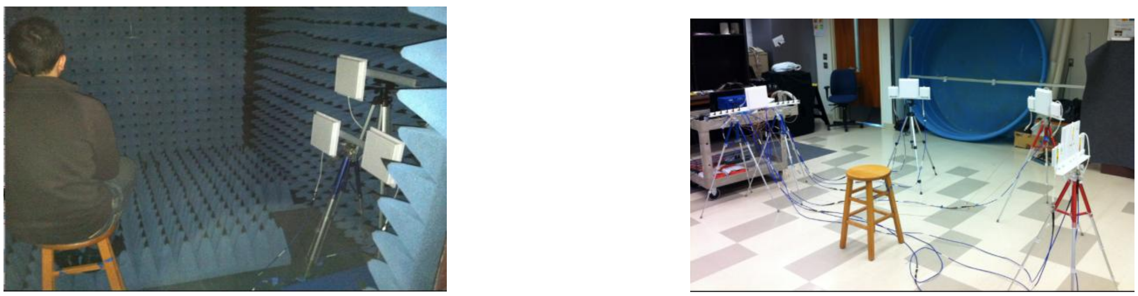
where, $J_n(\mu)$ is the *Bessel function of first kind* : $J_n(\mu) \approx \frac{\mu^n}{2^n n!}$,for small μ, the angle modulated signal contains all frequencies of the form $f_o + nf_b$ for $n = 0, \pm1, \pm2, \ldots$ and only the first sideband is important.

The respiration signal modulates the phase of the radar transmitted signal by a Doppler frequency shift due to a moving target, given by $\frac{2v_\tau}{\lambda_o}$, where v_t is the velocity of chest movement. The velocity of chest movement due to respiration is a periodic sinusoidal function, therefore,

$$\text{respiration signal model} = \text{Re}\left(A_c e^{j(\mu\sin\left(2\pi f_b t_{cpi}+\phi\right)+\theta)} \right) \qquad (6)$$

4 Experimental Setup and Results

Data used for this research project was obtained from 2 experiments performed in the lab. The first experiment was in the anechoic chamber (simulating ideal conditions) and the second experiment was a multi-static radar setup (took advantage of the MIMO properties of the STRADAR-16 receivers and 4 transmitters yielding 64 possible unique data sets from just a single observation) to observe the heartbeat and the respiration signals at different angles under real world noisy conditions.



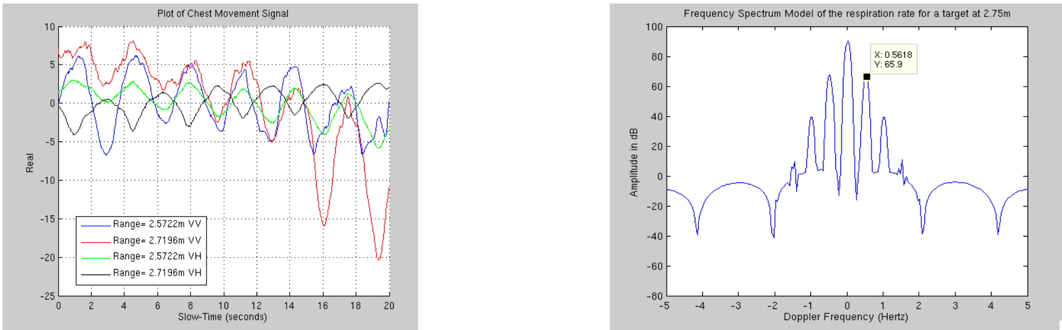
(a) anechoic chamber **(b) multi-static**

Figure 3: Experimental Setups

From the anechoic chamber experimental setup, the respiration and heartbeat signals under ideal conditions were obtained for 2 range bins and 2 different receiver polarizations (Figure 4 and 5). Figure 4(a, b) shows the ideal heartbeat signal (used in simulation) obtained when the human subject was holding his/her breath for at least 20seconds. Whereas, Figure 5(a, b) is the ideal chest movement signal containing both the heartbeat and respiration signals.

(a) **(b)**

Figure 4: Time and FFT plot of the Heartbeat Signal



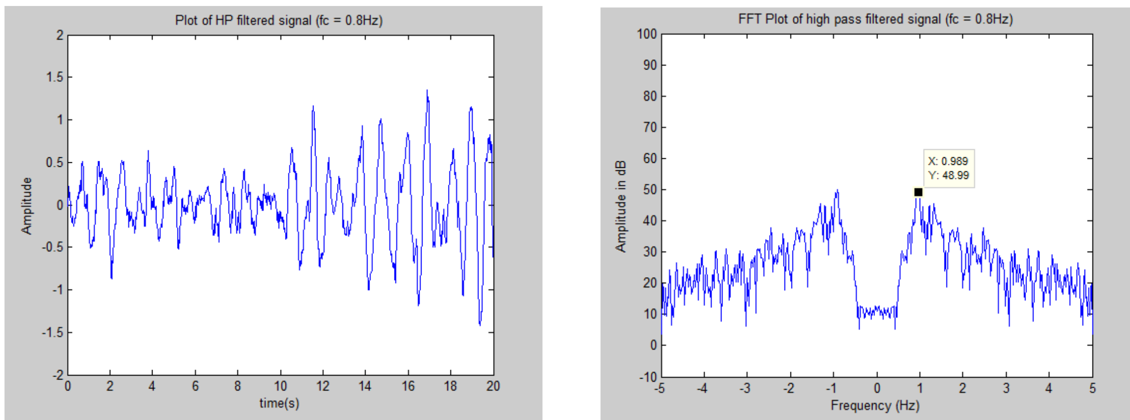
(a) **(b)**

Figure 5: Time and FFT plot of Chest Movement Signal

5 Algorithms Applied

5.1 Frequency Filtering

The heart rate frequency range is 0.8 - 2Hz, while the respiration rate frequency range is 0.1 - 0.8 Hz. The desired characteristics for any filter are low variation of the magnitude (ripple) in the pass-band, high attenuation in the stop-band and sharp-cutoff [18]. An IIR (with 9 coefficients) high pass Butterworth filter with a cutoff of 0.8Hz was implemented in MATLAB to separate the heartbeat and the respiration signals in the frequency spectrum. The Butterworth filter was chosen for 2 reasons: its flat pass-band with no ripple property and also ultimate roll-off towards zero on the stop-band. The behavior of the filter after cutoff was important because the heartbeat and respiration ranges overlap at 0.8Hz. The experimental chest movement signal (Figure 5) was high pass filtered using the Butterworth filter. Frequency filtering was unsuccessful mainly because the higher harmonics could not be eliminated from the signal. As a result the time plot of the filtered signal obtained was not identical to the heartbeat signal (Figure 4).



(a) **(b)**

Figure 6: FFT and Time Plots of a high pass filtered signal (Butterworth fc = 0.8Hz)

5.2 Time Frequency Analysis

Time frequency analysis [8, 15] was implemented for harmonic analysis by simultaneously observing time and frequency of the radar heartbeat signal. By using the Fourier transform (FT) a signal *f(x)* can be described using both its temporal behavior *f(x)* and frequency behavior *F(w)* (*w* is angular frequency and *x* is time), where *F(w)* can be obtained from *f(x)* using

$$F(w) = \int_{\mathbb{R}} f(x)\, e^{-2\pi i x w} dx \tag{7}$$

The heartbeat signal is a dynamic non-stationery signal and these properties cannot be captured fully by just using the FT because the time information is lost in the frequency domain.

The Short-Time Fourier Transform (STFT) is a method used for time frequency representation of a signal that combines the features of both *f(x)* and *F(w)* into a single function, *Vf (x,w)* (eqn. 8), which measures the amplitude of the frequency *w* at a time *x*. By fixing a window function, *g*, the STFT of a function *f(x)* with respect to g is defined by

$$V_g f(x,w) = \int_{\mathbb{R}^d} f(t)\, \overline{g(t-x)}\, e^{-2\pi i t w} dt, \quad \text{for } x, w \in \mathbb{R}^d \tag{8}$$

The spectrogram function in MATLAB was used to obtain the STFT of the heartbeat signal, therefore, enabling the analysis of the frequency components of the heartbeat signal at small time intervals.

The STFT was explored to verify if the frequency varying nature of the heartbeat signal with respect to time could be used to design an adaptive filter (based on time and frequency variations) to enable the separation of the respiration and heartbeat signals. The results obtained from the STFT (Figure 7) show that the frequency of the heartbeat signal at different time slots does not deviate by a significant amount. Therefore, adaptive filter design (based on time frequency variations) was not explored further.

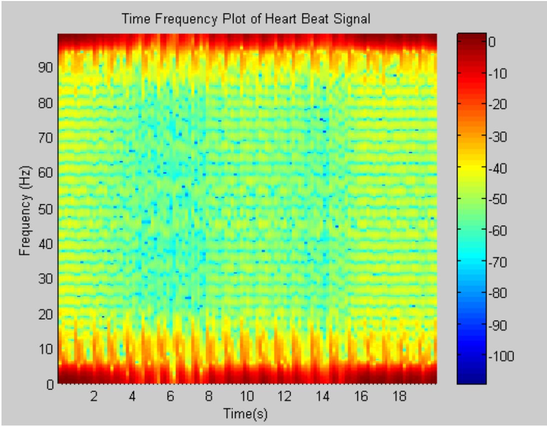


Figure 7: STFT Results: Time Frequency plot of heartbeat signal in the anechoic chamber

5.3 Blind Source Separation

Independent Component Analysis (ICA) is a statistical tool for separating data into informational components and can be used as a method for Blind Source Separation (BSS); source signals can be recovered from the data even if very little is known about the nature of those source signals. A common example of a situation where ICA can be applied to recover source signals is shown in Figure 8. If 2 people speak in a room with 2 microphones the microphones can pick up both the speech signals from the 2 speakers and the background noise signal from the crowd. The resulting output signal from each microphone is composed of a mixture of the 2 speech signals and noise. By applying ICA to mixed output signals of the 2 microphones the speech signals from both speaker 1 and 2 can be successfully recovered. ICA exploits that signals from different physical processes are *statistically independent* (the amplitude of the speech signal from speaker 1 at any given period of time provides no information regarding to the amplitude of the speech signal of speaker 2 at that time) [6,7,12]. In addition the number of different mixtures must be greater than or equal to the number of source signals [12]. The model for the ICA data is

$$x = As \tag{9}$$

Where **A** is an *m* x *n* unknown mixing matrix to be estimated, **x** is an *m* dimensional vector of the observed mixed signals and **s** is an *n* dimensional vector of statistically independent source signals.

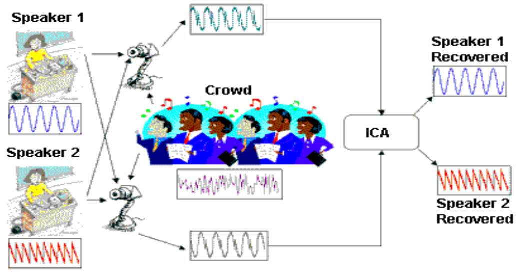
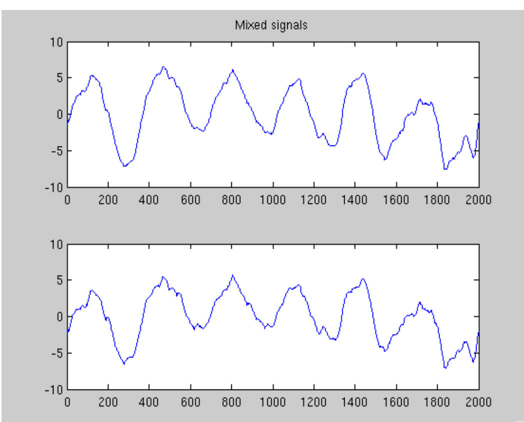
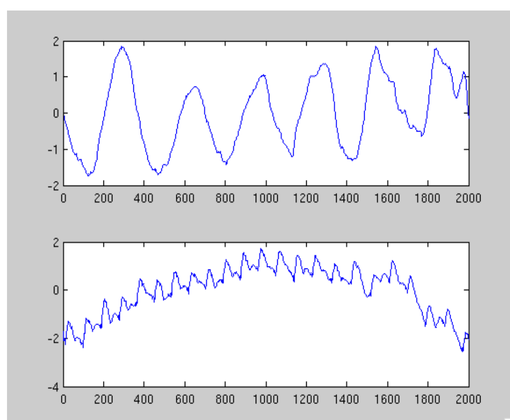


Figure 8: ICA example [11]

The MIMO (4 transmit channels and 16 receive elements) characteristic of the STRADAR were exploited so that ICA could be implemented; the amplitudes of the breathing and heartbeat signals vary when measured from the front or back of the human body [14]. Therefore, using the multistatic static experimental setup (Figure 3b), data from 2 receivers and 2 transmitters (positioned in front and back of the human body) was processed to recover the independent source signals (heartbeat and respiration signals). In addition a simulation was performed where the experimental heartbeat signal (Figure 4a) was mixed with an approximated respiration signal (eqn. 6) and the unknown mixing matrix was generated using random white Gaussian noise resulting in **x** (eqn. 9). The FastICA algorithm fundamentals that were proposed by Hyvärinen [14], yield a classic linear ICA algorithm with both good accuracy and fast speed of convergence. MATLAB FastICA GUI was applied to both the simulated and the experimental data in an attempt to separate the respiration and the heartbeat signals from the radar output signal. FastICA worked well on simulated data; heartbeat and respiration signals were successfully recovered from the 2 mixed signals (Figure 9). However FastICA failed to recover the heartbeat and the respiration signals when applied to experimental data.



(a)



(b)

Figure 9: FastICA simulation results - mixed and separated source signals

5.4 Linear Prediction

Since the radar output signal is a mixture of the respiration and heartbeat signals. To estimate and extract the heartbeat signal; the respiration signal function *y* is estimated using an autoregressive (AR) model defined by,

$$y_t = b_o + \sum_{k=1}^{p} b_k \, y_{t-k} + e_t \tag{10}$$

where:

y_t is the dependent variable which is a function of itself at the previous moments of time

$b_o, b_k \left(i = 1, \ldots, p \right)$ are regression coeffecients

e_t is a zero - mean white WSS process

p is the autoregression rank

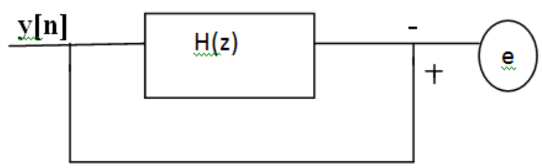
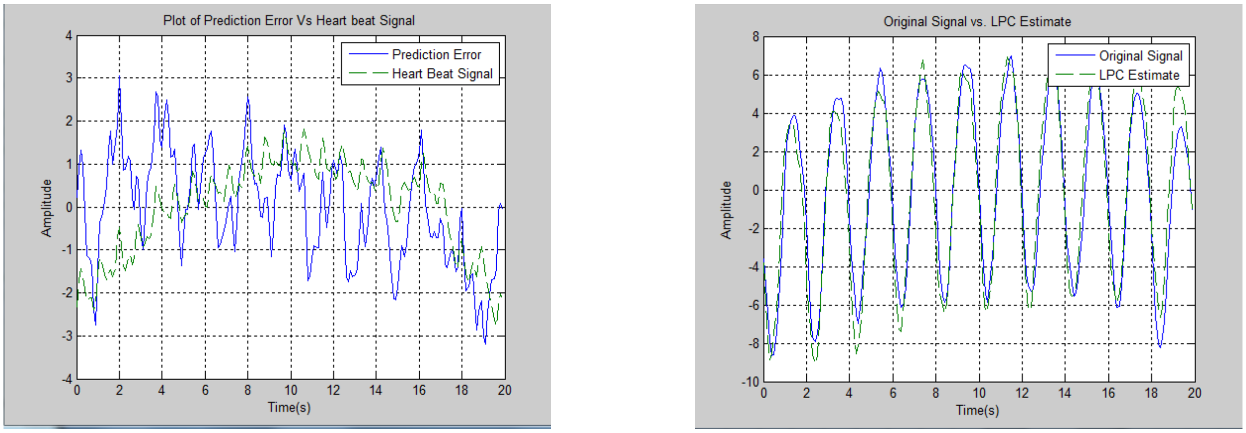


Figure 10: LTI system of an AR model

The AR model is used to estimate the respiration signal because the heartbeat signal is a weak narrow band signal and should not modeled using linear prediction on the radar output signal. By approximating the respiration signal using an AR model, the error signal, *e*, is expected to be the heartbeat signal. The regression coefficients, *b*, (eqn. 10) are estimated in MATLAB using the linear prediction function (LPC). LPC is used to estimate future samples of an autoregressive process based on a linear combination of past samples. An FIR filter *H(z)* defined by

$$H(z) = \sum_{k=1}^{p} b_k \, z^{-k} \tag{11}$$

is applied to the delayed samples of the original signal to optimally predict future samples [9]. Figure 10, shows how an AR model can be implemented using an LTI system. An algorithm for the prediction of *y[n]* (based on eqn. 10 and Figure 10) was developed and implemented in MATLAB. The algorithm was applied to a simulation that consisted of the experimental heartbeat signal mixed with the respiration signal model (Appendix B).



(a) **(b)**

Figure 11: LPC results – Plots of the LPC estimated and error signals

Figure 11a show that an AR model cannot approximate the respiration signal. Although, the prediction error signal has similar signal characteristics as the heartbeat signal the amplitude is higher. Since LPC failed in simulation it was not explored further.

5.5 Non-Linear Prediction

The maximum likelihood estimator (MLE) [17] is used to estimate the unknown parameters of the respiration signal model (eqn. 6). The radar output signal x is a mixture of the respiration s and the heartbeat signals, where the heartbeat signal is masked by the respiration signal,

$$x = s + \mathrm{n} \tag{12}$$

where:

$$s = \mathrm{Re}\left(A_c e^{j(\mu \sin\left(2\pi f_b t_{cpi} + \phi\right) + \theta)} \right)$$

n is zero-mean white Gaussian noise with variance δ^2

By using a Gaussian random variable the PDF of x is

$$P_x(X \mid s, \delta^2) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(x-s)^2}{2\delta^2}} \tag{13}$$

where:

s is the mean

δ^2 is the variance

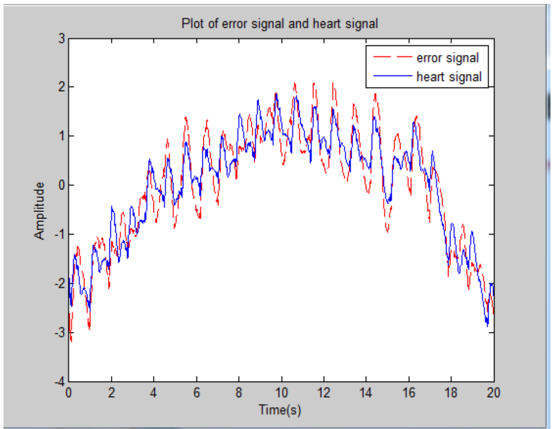
The likelihood function

$$L(A, m, f, \theta, \phi) = \prod_{n=1}^{N} P_x(X_n \mid s, \delta^2) \quad (14)$$

can be determined by taking N samples of data from the radar output signal. In addition, by taking the gradients of the likelihood function

$$\begin{bmatrix} \frac{\partial}{\partial A} \log L \\ \frac{\partial}{\partial m} \log L \\ \frac{\partial}{\partial f} \log L \\ \frac{\partial}{\partial \theta} \log L \\ \frac{\partial}{\partial \phi} \log L \end{bmatrix} = 0 \quad (15)$$

The 5 unknown parameters of equation 6 can be determined from the 5 equations above and s can be estimated. Subtracting s from the radar output signal x results in an estimate of the heartbeat signal. MATLAB's least-square optimization toolbox was used to estimate the 5 unknown parameters and this is equivalent to solving equation 15. The MATLAB algorithm was applied to a simulation that consisted of the experimental heartbeat signal mixed with the respiration signal model (Appendix B); the results obtained are shown in Figure 12. From Figure 12 it is evident that MLE worked perfectly in simulation since the heartbeat signal was recovered from the radar output signal. However, non-linear prediction failed when applied to experimental data.



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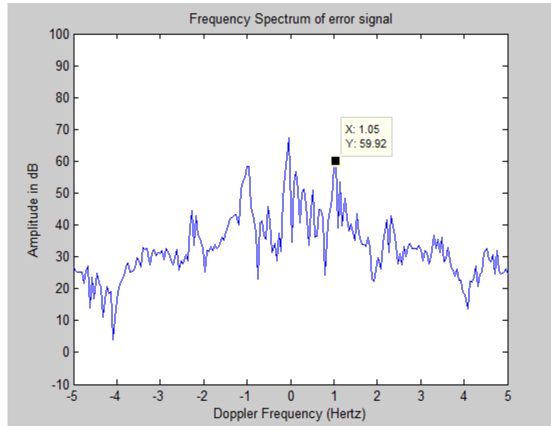

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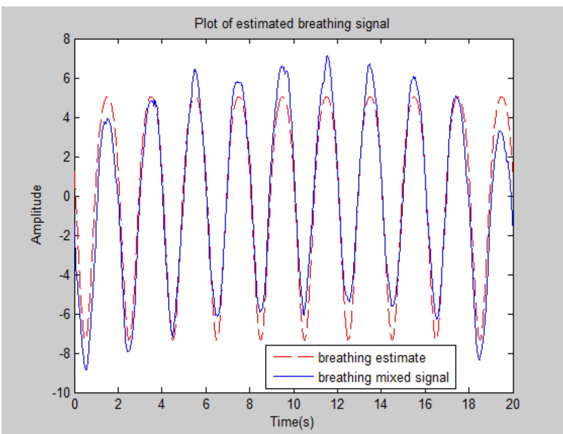


(c)

Figure 12: MLE Results

6 Conclusion

A model for the respiration signal has been developed (eqn. 6) and tested in simulation with the signal processing techniques explored in an effort to recover the weak heartbeat signal from the chest movement signal. The five signal processing techniques that have been explored and implemented in MATLAB are: blind source separation (BSS), linear prediction, time frequency analysis, frequency filtering and non-linear prediction. The non-linear prediction and blind source separation techniques have yielded the best results with simulated data as shown in Appendix B. That is in other words, heartbeat and respiration signals have been successfully separated from a simulated radar chest movement signal. However, when applied to experimental data, non-linear prediction and BSS failed to recover the heartbeat signal.

The heart rate frequency range is 0.8 - 2Hz but the respiration rate frequency range is 0.1 - 0.8 Hz. Thus, high pass frequency filtering was implemented but was unsuccessful because the higher harmonics could not be removed from the filtered signal. Therefore, time frequency analysis which provides an analysis of harmonics was explored. Unfortunately based on the time frequency analysis of the heartbeat signal it was discovered that there was very slight frequency variation of the heartbeat signal with respect to time. As a result adaptive frequency filtering was not explored. The respiration signal model (eqn. 6) was estimated using an autoregressive (AR) model and the coefficients were obtained using linear prediction coefficients; the results show that an AR model cannot be used to estimate the respiration signal.

Although blind source separation(FastICA) and non-linear prediction successfully recovered the heartbeat signal from the simulated data, further work is needed to recover the heartbeat signal from the experimental chest movement signal. The algorithm developed for non-linear prediction in MATLAB can be improved to achieve accurate initial bounds used in the estimation of the unknown parameters from the respiration signal model.

7 Appendices

Appendix A: Obtaining the respiration signal in MATLAB

Radar Data Cube processing:

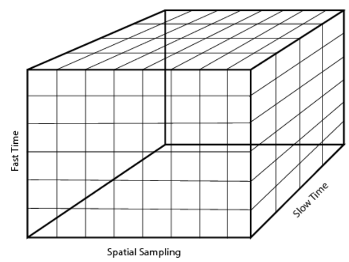


Figure 13: Data Cube processing [10]

To obtain the range and the Doppler frequency of the corresponding respiration and heartbeat signals, further digital processing is implemented in MATLAB. The data received from the STRADAR system is received by a PC through a USB connection. The received data consists of L samples of each PRI (fast time), N receiver elements and M pulses (slow time); a data cube of an L x M x N matrix is formed in MATLAB (Figure 13).

Fast-time processing:

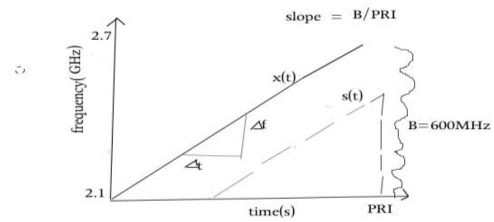


Figure 14: Obtaining range from fast time

An L x 1 x 1 vector from the cube corresponds to fast time (where L are the number of samples per pulse). The fast time vector is the first term in equation 5,

$$z(t) = \mathrm{Re}\left(0.5Ae^{j\left(\frac{2\pi\beta\tau t}{Tr} + 2\pi f_o\tau - \frac{\pi\beta\tau^2}{Tr}\right)} \right) \tag{16}$$

Taking the FFT of the fast time vector allows the range of a target to be obtained, where Δf corresponds to the frequencies from the FFT. Based on Figure 14, range can be obtained using,

$$Range = \frac{PRI \times c \times \Delta f}{2\beta} \tag{17}$$

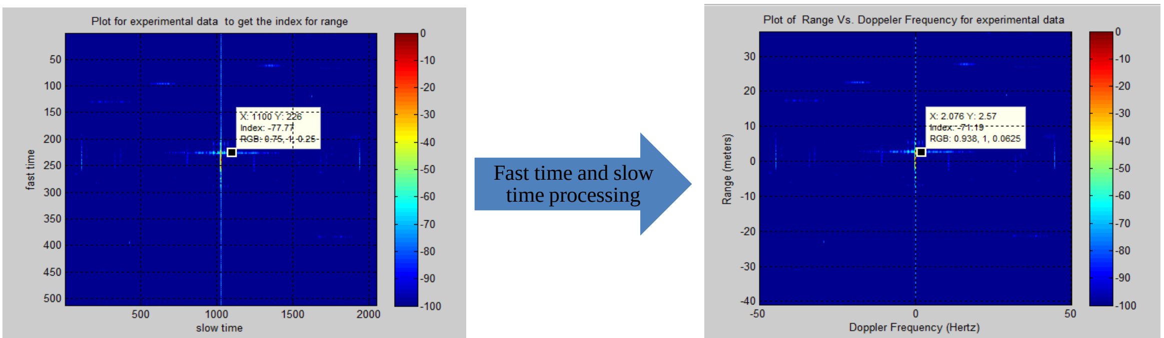
Once the range bin of the target has been established, the corresponding Doppler frequencies can be obtained by processing the slow time vector.

Slow-time processing:

By extracting the 1 x M x 1 vector, where M corresponds to the total number of pulses for each CPI, the slow time vector can be obtained. Taking the FFT of the slow time vector yields the Doppler frequency for a particular range bin. The equation for obtaining Doppler frequency is

$$Doppler \quad Frequency \quad Shift = \frac{2v}{\lambda} \tag{18}$$

By combining both the slow and fast time processing techniques and picking only 1 receive element, the L x M x 1 matrix can be converted to obtain both Doppler frequencies and range plots. Figure 15 shows a plot that is obtained. From this plot, the corresponding respiration or heartbeat signals can be obtained by picking the range bin of the target.

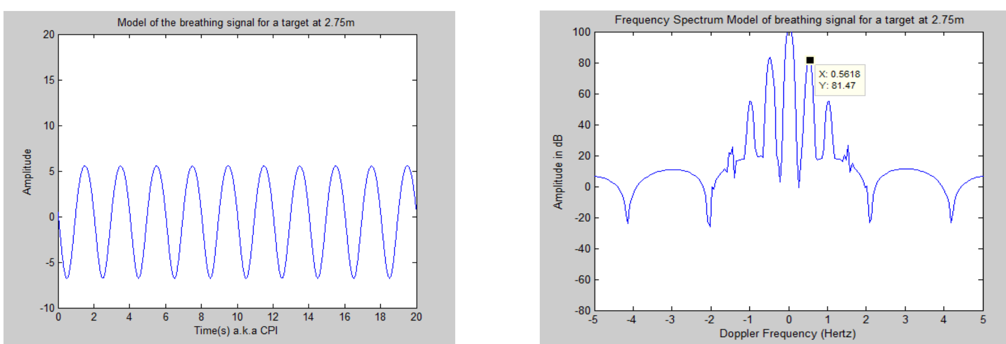


(a) **(b)**

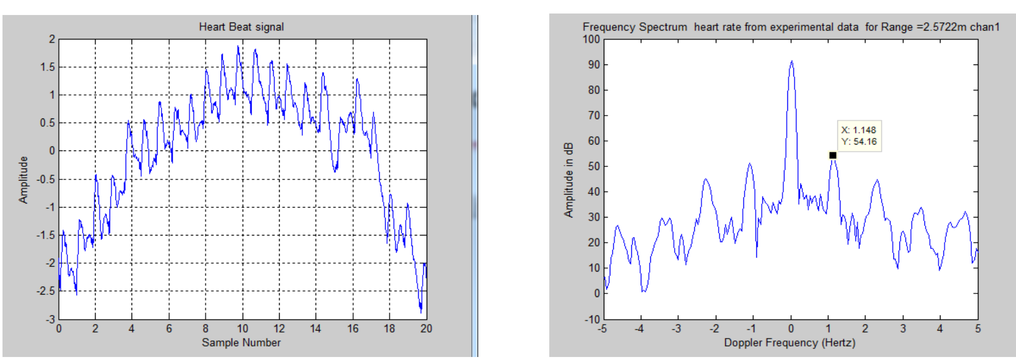
Figure 15: Obtaining range and Doppler frequency from the data cube

Appendix B: Signals used for Simulation

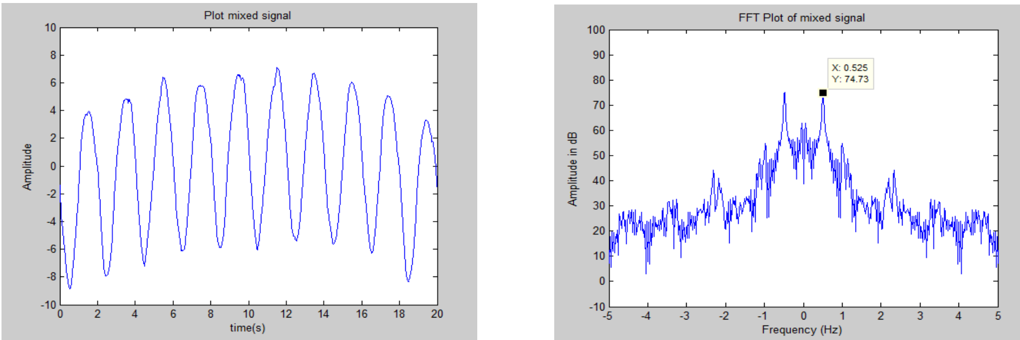
For simulation, the chest movement signal is the sum of the respiration model (eqn. 6) and the heartbeat signal from the anechoic chamber resulting in Figure 16(e) below.



(a) **(b)**



(c) **(d)**



(e) **(f)**

Figure 16: Simulation of radar chest movement signal using the respiration model (eqn. 6)

8 Aknowlegments

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